



History of Game Theory: A Review

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Abstract

Game theory, as an interactive many Nobel Prize winning decision making theory was first formally introduced by John Von Neumann and Morgenstern in 1944 in the name of “Theory of Games and Economic Behaviour.” Publication of theory of games and economic behaviour generally regarded as a starting point of Game Theory and came in its own scientific discipline. Game Theory as a new area of interest in research on theory and application in economics and many other disciplines including biological sciences. It has a range of its own historical development and growth for promoting itself as a scientific disciplinary direction. Based on this understanding this paper attempts the review the historical nature and development from its prehistory period to modern times and it also looks its development as a theoretical analysis of modelling in social sciences and other areas in history. It could represent how the formal structure of Game Theory over this period had developed the form and code of human decision and behaviour.

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1. Introduction

Game Theory, as an interactive decision theory, was first formally introduced by John Von Neumann and Morgenstern in 1944 in the name of "Theory of Games and Economic Behavior". Publication of "Theory of games and Economic Behavior" generally regarded as the starting point of game theory that has its own scientific discipline. This book has become a fundamental new grounds in several directions. The formal theoretical analysis was developed by Augustin Cournot in 1838. Later in 1921, Emile Borel a mathematician who had given an idea about formal notion of games. In 1928 John Von Neumann a mathematical theoretician student of Hilbert, who published independently a paper on parlor games in a journal of Annals of Mathematics. After that Von Neumann who associated with Oscar Morgenstern, an economist, both of them framed a full-fledged game theoretical model for the analysis of human social behavior of any interactive situation on decision making in humanities. It was in the hands of John Nash (1950), a Nobel prize winner, who demonstrated a finite game that always has the equilibrium point, which means that players chosen the action best in response of the belief with given opponents. Based on this historical development of game theory, in this article an effort has been made to look into the game theory in its historical development as a theoretical analysis of modeling in social sciences and other fields. It could represent, how the formal structure of game theory over the period in history had developed in the form and as a code of human decision and behavior. It also gives a modern time development of Game Theory.

2. Origin of the Game Theory:

The strategic games have long been a prehistory. The art of war written by Sun Tzu in the third century BC or earlier represented notion of war as a zero sum game or constant sum. Emerson Niou and Peter Ordshook (1990) credit Sun Tzu about dominant and mixed strategies and understanding of minimax strategies. Concept of mixed strategies based on the concept of probability was indispensable to reach some interesting results. The beginning of the probability calculus was associated with the correspondence of Pierre de Fermat (1607-1665) and Blaise Pascal (1623-1662) dated in 1664 (Hyksova, M.), James Waldegrave (1684-1741) provided the first known mixed strategies solution of a matrix game type. It was related

to the game Le Her and described in Waldegrave's letter to Pierre Remond de Montmort(1678-1719)dated on Nov 13, 1713.The game Le Her was already investigated by De Montmort and Nicholas Bernoulli(1687-1759)in their mutual correspondence in 1713.JamesWaldegrave was looking for a strategy that maximizes the probability of player's win exactly in the sense of today minimax principle.Waldegrave solution remained almost unnoticed for a very long time (Hyksova, M).Hence, true prehistory was started only after this event and took 350 years to represent the first key mile stone in the history of the game theory.

Game theory in the game format was first inventedvery formally in order to satisfy a mathematical curiosity (Schmidt 1995). Game Theory was to try to find out a theoretical solution to the problems posed by uncertainty in a game of chances. In the beginning of the eighteenth century,Cournot (1838) - a well-known economist and theoretician - was framed and constructed the notion of game theory in economics in the form of oligopolistic firm that includes monopolists and perfect competitors as limiting extreme.He developed game model of oligopolisticcompetition. He was also on the verge of application of Nash equilibrium model in this oligopolistic theory of competitive behavior (Myerson 1999).Cournot wrote a book on mathematical economics first analyzed competition among firms that compete to sell the same consumergood. He was not made any attempt to articulate a general methodology of equilibrium analysis (ibid).Cournot's method of analyzing the game form did not look like any compelling general theory of a rational behavior in the competitive firm. But he has given a formal idea of a game format of competitive behavior between the firms.

Later, E.Zermelo (1913) a German mathematician who first formulated and completed strategies of games. It was generally agreed that the first formal theorem in the theory of game was proved by E.Zermelo in an article on Chess appearing in German in 1913.Some writers(Aumann 1989, Eich Berger 1993, Hurt 1992) claimed that Zermelo is showed chess is determinate.Others claimed that Zermelo's method of proof was by backward induction (eg. Binmore, K 1992).Backward induction is referred in the game theory as it requires starting from the end of the game and then working backward to its beginning.Zermelo's proof can be bridged using zermelo's original idea of non -repetition of positions. This is in contrast to DenesKonig's (1884-1944), a Hungarian mathematician, who conjecture that a proof of Zermelo's theorem first requires proving the boundedness of the

number of moves. Zermelo, and other game theoreticians like Konig and Kalmar's (also a Hungarian Mathematician -1905-1976) shows that what we now called the two person zero sum games with perfect information. The common starting point for their analysis was the concept of winning position, defined in a precise mathematical way (Schwalbe and Walker 1999). In contrast of the work of Zermelo, Konig and Kalmar, Von Neumann's main concern were the strategies interaction between players and the concept of equilibrium. These two ideas were the building block of modern non-cooperative Game theory (ibid).

The next exponent in this field of game of theoretical enquiry was developed Emile Borel (1871-1956), a German mathematician published a series of notes on symmetric two players zero-sum games with a finite number of pure strategies of each player (Myerson, M). Emile Borel (1913) introduced the concept of "method of play" in the sense of today "pure strategy" and looking for a solution in a mixed strategy in the sense of minimax theorem. In his 1921 paper, Borel proved that the existence of a solution for the three persons' game. Later he proved the same solution for five-person games. In 1953, M. Frechet published an article in *Econometrica* about the English translation of Borel notes and designated Borel as the initiator of game theory. Borel set out "to investigate whether it is possible to determine a method of play that is better than all others" while laying out the formal structures of his model. He further remarked that a "Method of play should be understood here to mean a code that determines for every possible circumstances (support of finite number) exactly what the person do" (Myerson 1999). Having made this remark, Borel felt that to ignore the extensive temporal structure of games (ibid). Borel simply represented each game a matrix of number that specifies the expected value for each player for each pair of such methods of play. Von Neumann (1953) gave a prior credit to Borel for the basic concept of a strategy. It is hard to see how economists could have learned the principles of general strategies independently from Borel's remark (Myerson 1999).

3. Von Neumann and Osgor Morgenstern Model of Game Theory:

John Von Neumann an outstanding mathematician, Game Theorist, student of Hilbert independently published in 1928 an article on Parlor games. Von Neumann born in Budapest, Hungary, in 1903. When he studied at University of Budapest (1921-23), he made contact with David Hilbert, at Gottingen, the Mecca of German Mathematician and walked close to

him on such topics as set theory and foundations of Mathematics, Hilbert space theory, Operator Theory, and The Mathematical foundation of Quantum Mechanics. Von Neumann received his doctorate in Mathematics from Budapest in 1926. Until his death in 1957 his involvements were wide ranging from continued to work on mathematical physics and extensive military consultancy on Weapon development including work of the Atomic bomb at Los Alamos to foundational research on digital computing and cellular automata (Leonard J. R. 1995).

Von Neumann turned his attention to the mathematical analysis of games and produces his minimax theorem in 1926. In 1928 Von Neumann published three papers on set theory. His attention on set theory followed Hilbert construction on axioms of Euclidean geometry, in which he showed which groups of basic postulates were necessary for the competitors and consistency of that system (ibid). According to fellow Hungarian, Nicholas Kaldor, Von Neumann interested in economics in 1927. Based on Kaldor's suggestion he spent time to read Kurt Wick sell's book on "Value, Capital and Rent" which provided an introduction to Walras and utilized Bohem-Bewerks on Capital Theory. In this, he immediately criticized Walrasian system and observing that it permitted negative values (ibid). His idea on economics differed many other economists of those periods. He was viewed as a part of Neo classical general equilibrium group associated with the Menger collegium in economics. In November 1939, Von Neumann listed the theory of games as possible topic of his lectures at University of Washington and mentioned unpublished material on Pocker (Dimand and Dimand 1995). Even before the meeting, Morgenstern in Princeton, Von Neumann were aware of his minimax theorem was relevant to economic theory (ibid). Von Neumann pointed out that in a game each player must choose his strategies without being informed of the other's players strategy choices. Based on his game model he argued that any competitive game can be modeled by a mathematical game with the following simple structure. He means that there must be set of players, each player has a set of strategies, each player has a payoff functions, and each players' choice of his strategy should be independent. Based on this structure he calls this type of game is normal forms of game.

Von Neumann (1922) assumed that players would not simply choose their strategies independently, but would coordinate their strategies in coalition. He also assumed that

players are transferable and that all the games are zero sums. In his minimax theorem Von Neumann (1928) showed that the general existence of minimax solution in randomized strategies for finite two players zero-sum games. For such a game, the minimax theorem is logically equivalent to the existences of Nash Equilibrium (Myerson 1999).

Oskar Morgenstern was another exponent involved in establishing theory of game and collaborated with Von Neumann both were to be published the monumental work of "Theory of Games and Economic Behavior" in 1944. Morgenstern born in Silesia of Germany, in 1902 moved at age of 12 with his family to Vienna. In 1925, he obtained his doctoral degree at the University of Vienna with his thesis on Marginal Productivity. Succeeding Friedrich Hayek in 1931 as director of Vienna's Institute for Business Cycles Research, he held that position until Germany occupied Austria in 1938 in periods of World War II. He was moved to US Princeton University; soon he became a permanent faculty member. Later in 1970 he moved to New York University where he remained to be a faculty member until his death in 1976.

Basically Morgenstern was not a mathematician but economist. In the 1930's Morgenstern attended the mathematical colloquium of Karl Menger and was tutored in mathematics by Abraham Wald. Morgenstern was presented his paper on "Perfect foresight and economic equilibrium" in 1935 expanding the problem of strategic interaction. He published several books and articles of which his "The Accuracy of Economic Observations" in 1950 is perhaps the most well-known.

The Czech Mathematician, Edward Czech drew attention of Morgenstern to Von Neumann on game theory. After meeting John Von Neumann at Princeton, Morgenstern engaged him in the long and fruitful conversation about games. Both were collaborated and initially expected to produce a paper for submission to the Journal of Political Economy, then a pamphlet, then a short book and finally a book of well over six hundred pages on "Theory of Games and Economic Behavior" in 1944 (see Morgenstern 1976, cited in Dimand and Dimand 1995).

Von Neumann and Morgenstern elicited to use a method of axioms, definition and successive refinement in Game Theory. This is the novel approach in economics led them to deal more carefully and explicitly with such issues as the definition of solution and or games

of information structure (ibid) The theory of games becomes more than a response to Morgenstern's earlier call for a dynamic theory of interdependent economic decisions. It becomes a part of general shift in science which involved, broadly speaking the abandonment of determinism, continuity, calculus and the metaphor of the "Machine" to allow for indeterminism, probability and discontinuous changes of State". The mechanical language of the calculus capitulated that what Weyl called "Arts Combinatoria".

The outstanding period of this publication of "Theory of Games and Economic Behavior" in 1944 by John Von Neumann and Oskar Morgenstern came into its own scientific discipline. This book broke the fundamental new grounds in several directions. This includes the notion of co-operative game, its conditional form and its Von Neumann's and Morgenstern stable sets (Aumann 2008). Both of their work was the basis for postwar research in Game Theory. Initially it was a specialized field on application of military strategy and statistical decision theory, eventually permitting to industrial organization and public choice and influencing macro-economics and international trade. Most economists had to rely on Leonard Hurwitz, or Herbert Simon, Richard Stone or Abraham Wald or another reviewer, for a sense of understanding of what Von Neumann and Morgenstern had been achieved and proposed (Dimand and Dimand 1995).

4. John Nash Equilibrium and its Demonstration:

In November 1951, John Nash first paper, on Non-Cooperative Equilibrium, received much attention among the game theoreticians, economists and others. Four articles published by Nash between 1950 and 1953 are the starting point of refinement in game theory distinct from "Theory of Games and Economic Behavior" of Von Neumann and Morgenstern volume. The formulation of Nash Equilibrium has had a fundamental and pervasive impact in economics and other social sciences which is comparable to that of discovery of the DNA double helix in the biological sciences (Myerson 1999). For Nash axiomatization and modeling must be understood as two complementary approaches. Nash first contribution was his Theory of two people bargaining. Nash introduced in 1950 a bargaining solution. It was the first work of Game Theory that did not assume transferable utility. Most subsequent work on cooperative games with non-transferable utility has been based on Nash's approach to the bargaining problem. Nash bargaining theory builds on the insight that individual's utility

scales can be defined up to separate increasing linear transformation but this result follows only from Von Neumann and Morgenstern's 1947 deviation of utility. Most of the Nash thesis was published in the Annals of Mathematics. Nash (1951) presented a number of interesting examples, illustration problems that have concerned with game theorists ever since, including a game with one Pare to-inefficient equilibrium like the Prisoner's dilemma, a game with multiple equilibrium and a game with unstable equilibrium that shows a need for refinements such as perfect equilibrium. Nash (1951) also analyzed a three person Pocker game in extensive form when he applied Kuhn's (1950) new methodology of studying behavioral strategies in which randomization occur at each strategies of the game rather than the mixed strategies of Von Neumann.

Nash important contribution was his argument that thenon-co-operative equilibrium concept together with Von Neumann's normal form, give us a complete general methodology for analyzing game (Myerson1999). Nash applied normalization argument to show that any other theory of games should be reducible to equilibrium analysis. Nash equilibrium is our general solution concept (ibid). Nash (1951) dropped the assumption of transferable utility and dropped that zero sum restriction that Von Neumann imposed. In 1953 paper by Nash offered an application of his program for reducing cooperative game theory to non-cooperative equilibrium analysis. He modeled the two person bargaining process by a simple game of simultaneous demands. The Theory of non-cooperative game that Nash found has been developed into practical calculus of incentives that can help to better understand the problems of conflict and cooperation in virtually any social, political or economic institution. Nash theory of non-cooperative game now become recognized as one of the outstanding intellectual advances in the twentieth century and after. Nash solution favors the equilibrium as against the rationality feature of the solution. Infact, his solution is defined in terms of the equilibrium property(Hurwitz). Nash definition yields the same saddle point solution that is implied by the Von Neumann and Morgenstern solution concept that corresponds to both players maximizing. Arrow and others are pointed out that Nash solution when applied to the Classical Oligopoly problem essentially corresponds to the so called "Cournot solution".

5. Game Theory Structure and conceptual Prelude:

Game theory is the formal study of conflict and cooperation. Game theory concepts apply whenever the action of several agents is interdependent. These agents may be individuals, groups, firms or any contributor of these. The game theory provides concept as a language to formulate structure, analyze and understand the strategic scenarios. These concepts are in the historical context which has been developed over the periods of time and many contributors who had been involved under these processes.

Game theory studies the interactive situation of agents which means the outcome of the agents' action depends on the action of all other agents involved are called "interactive". The fundamental idea is that an agent has an interactive decision should and does take into account the deliberations of the other players involved, who, in turn, take the deliberations into account. Under this, game theory relied heavily on mathematical model in explaining the structural epistemic issues concerning modeling and unreasonable assumption in philosophy of economics also relevant. (Grune T and Aki Lehtinen 2010)

During the earliest years, Game Theory was pre occupied with strikingly a competitive games-known as two persons zero sum games. In these games no point in cooperation or joint action of any kind, preference is revealed necessarily (Aumann 2008). The extensive (or tree) form of a game is consisting formal description of how game is played, with players move, their knowledge, chance occurred, the payoff at the end. This is introduced by Von Neumann (1928). The extensive form of game later generalized by Kuhn (1953). Later it has enormous influential for beyond zero sum theory.

Application of equilibrium and optimization in game theory was the foremost conceptual discussion in the literature of game theory. Borel (1913) assumed that a game had been solved when players maximized probability of winning. For Von Neumann and Morgenstern "solution" depends on dominance –players ruling out strategies which would definitely disadvantage them. The application of dominance depends on the objectives of players and the rules of the game played. This definition solution applied the problems of individual's cooperative games, games of tidily wins and games of politics (Dimand and Dimand 1995).

Von Neumann and Morgenstern stressed that coalition formation is permitted in the form of game where individual could benefit from it. Hurwitz (1945) noted that possibilities of duopolists forming a coalition. Here no rigorous theory was developed under this game theory that has made a real progress. Coase R.H (1937) considered that formation of coalition (organizations) might be more efficient than market contracts.

Game theory remembered with minimax strategy as player's objectives stemmed naturally from Von Neumann and Morgenstern's emphasis on zero sum game which arose from the concern with gambling by precursors in game theory. In such games i.e., A's loss is B's gain. Under this situation, it is complete conflict and maximizing the minimum payoff one can achieve if one's opposite's plays in a hustle fashion is quiet reasonable (ibid). Solution derived from minimax objectives were a subset solution as defined by Von Neumann and Morgenstern on choice problems with a high degree of interdependence between agents were chiefly concern with games in which there was uncertainty. In 1928, John Von Neumann stated that minimax theorem for the two person zero sum game with finite numbers of pure strategies and constructed first valid proof of the theorem using topological approaches based on Brouwer's fixed point theorem.

The juxtaposition of game theory will formalize several observations on the evolution of distinction between cooperative and non-cooperative games of most general analytical importance. The non-cooperative game theory is concerned with the analysis of strategic choices. The paradigm of non-cooperative game theory is the details of ordering and timing of player's choice is crucial to determining the outcome of a game. Anon-cooperative game theory model of bargaining would pose a specific process in which it is pre specified who gets to make an offer at a giventime. The term non-cooperative means thatthis is a branch of game theory explicitly models the process of players making choices out of their own interest (Turoy and Von Stengel 2001). The strategic form (which is the normal form) is the basic type of game studied in non-cooperative game theory.

On the other hand, a coalitional or cooperative form of game is high level of description. It specifies what payoff of each potential group or coalition can obtain by the cooperation of its members. It investigates coalitional game with respect to the relative amounts of power held by various players or how a successful coalition should decide its

proceeds. This concept of game is mutually applied in political science and interactive relation. In a cooperative game theory Nash proposed a solution for the division of gain from agreement in a bargaining position. The amount of power a side has is determined by usually inefficient outcome that results when negotiation breaks down. Nash Model fits within its cooperative frame work. It favors outcome of the bargaining process.

The concept of equilibrium in game theory is an outstanding formula in the interactive decision of game format. Generally, optimization or maximization in the interactive process of players is to be considered as outcome in the game process but attainment of equilibrium become is shift in the examined process through reasoning of players able to stimulate other's intended strategies by leaning on a common knowledge of game characteristics. Nash equilibrium is to be the adequate equilibrium notion in the game theory.

6. Game Theory in Economics and Other Social Sciences:

Game theory emerged as a new branch of economic analysis a long time after 1944. A Complete understanding of the relationship between game theory and economic theory requires investigating the nature of filiations from Von Neumann to Nash Works. In the Theory of Games and Economic Behavior we find the lines of a precise research program. This part of the program is axiomatization of theory of utility, definition of a concept of solution, demonstration of the existence of a solution in the case of zero –sum two-person game (Schmidt 1995). During the 1960s and early 1970s economists began to give their attention to a new set of prediction for which game theory seemed to be new ultimate modeling tool.

Von Neumann mathematical thought to economic theory are more fragile and partially contingent (Schmidt 1995). The applicability of strategic games, in the sense of his 1928 article is obviously not limited to the domain of economics. The connection between the minimax theorem and the saddle point is the result of a property of convexity. Independent of any economic interpretation of it that might be given. Two thirds of theory of games and economic behavior are directed to zero sum games and non-zero sum games are handled with recourse to the devise of the ‘fictitious Players’. Over the course of twenty

years' game theory was an object of study or a small group of mathematicians or research tool for military strategies.

The application of game theory to economics posed a more fundamental problem due to distance separating several major concepts (rule of game, game solution, coalition etc.) from the categories constructed by economic analysis. There are small group of economists try to bring game theory format in economic analysis by focusing themselves the hypothesis of transferability of utilities. Introduce dynamics into static treatment of interaction between the players abandon inelastic frame work of complete information. In the initial excitement it seemed that the theory promised to place all of economic theory on a totally new strategic floating in which the way concept of the "competitive" process would be restricted and rephrased in terms of games of strategy.

The publication of Nash article in 1950s refreshed for reconstruction of refreshed thinking in game theory. Shubik (1955) rediscovered in Cournot's work for premises of Nash's concept of equilibrium (Schmidt 1995). Harsanyi compared Nash model of negotiation with economic analyses beginning with Zeuthen and continuing with Hicks. Not sure that several problems posed by application of game theory to economics were resolved in the 1960s. Most active research economic roots of game theory are the work of writers of Shubik, Harsanyi, Shapley and Aumann.

In the end of 1970s the connection between game theory and economics entered a new phase. Game theory approach invaded progressively in many sectors of economic analysis –on Industrial economics, Insurance economics, monetary economics, and financial economics, and part of international economics further with the economics of law. Now full-fledged game theory with micro economics. There are proportionally fewer pure mathematicians are working on game theory. Economist feel comfort to bring closer game theory to the economic environment. Divergent of explanation emerged among economists. The relation between game theory and economic science is in the process of relishing itself.

Writers in economics often tended or tend to equate solution with competitive market clearances, although model of monopoly, oligopoly, and collusion had been discussed frequently and informally since Adam Smith and more formally beginning with Cournot (Dimand and Dimand 1995). Under this Von Neumann and Morgenstern were the

first writer to define a concept of static economic equilibrium that did not depend on limiting the form of interaction modeled to perfect competition or ideal to markets. Unless previous treatment of equilibrium such as general competitive equilibrium of Walras, Pareto, and Fisher, Von Neumann and Morgenstern's definition of equilibrium did not depend on any particular "rules of the game" although any application of the concept in model –dependent.

Game theory in other sciences floored to give a more an analytical tool as a main exponent of working with new insights. In 1954 Lloyd Shapley and Marin Shubik published paper on which represent one of the earliest explicit application of game theory in Political sciences. Many other works were appeared by R.D. Luce, A.A. Rogow from 1956 onwards. W.H. Riker who had been published on political matters from 1959. Soon the game theory found crucial place in political sciences. Many of the modern monographs on political sciences regard game theory as an inseparable part of this discipline.

7. Game theory in Modern Times;

Game theory in Modern times emerged is to fulfill its extension form both analytically and theoretically to compensate to be in application front in economics and other social sciences. It is also evolved in the biological sciences as the main tool for the investigation of conflict and cooperation of animals and plants. (Hykova, M.) The Logic of animal conflict by Maynard Smith John and G.R. Price published in 1973 stimulated a great deal of successful works and application of game theory in evolutionary biology.

Many developments in game theory are taken place in modern times. Its nature processes of defining game structure, application of complete information, and incomplete information, common knowledge etc., with cooperative and non-cooperative games and many other conceptual fields. Non cooperative game theory was applied to large variety of particular economic model. And this led to the study of important variants on the refinements of the equilibrium concept (Aumann 2008). New lights were shed on old concepts such as randomized strategies. After the International journal of game theory was founded in 1972, Game theory has matured (ibid).

In analyzing specific economic models using the strategic equilibrium an activity carried with great vigor since 1975. Various refinements of strategic equilibrium have been

defined. Recently the theory of automata has been used for formulations of bounded rationality in repeated games. The literature on the game theoretic approach to cost allocation is also quite large. Thus game theory become revolutionised in its own theoretical defining process and caliber of its application of many other fields other than in economics. Game theory is quite large enough to understand in the field of studying the order of epidemics and its process with which high intend of analytical variation in application of many fields.

8. Summary and Conclusion

In the word of Aumann, one can understand what the game theory so far in its history telling us the nature of human mind and the behavior have the integration in overhead as game form. It is said that Game theory is a tool for telling us where incentives will lead. Also that game theoretic results developed in one context often have important implications in completely different contexts. Alas game theory is a method and a methodological program to be explored an underpinned epidemics demystify to fore and categorize the economic science and other disciplines to have a valid result oriented field for research.

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