



Optimization and Analysis of a Counter-Flow Dehumidifier with a Thermo-Electric Heat Pump

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Abstract

A generic thermoelectric couple was investigated to optimize for optimum water extraction and dehumidification of humidified air. The temperature of the cold face of the thermo couple was kept lower than that of ambient temperature of pumped air so that temperature gradient is formed on the cold side. This mechanism saturates and condenses the incoming air on the cold surface of the thermoelectric couple. The heat (Q_L) at temperature T_{SC} , accumulated on the cold side is pumped to the hot side at temperature T_{SH} . The heat, Q_H , on the hot side must be continuously expelled by the electric power provide by the thermoelectric couple. To accomplish this, a counter flow cool air is fed to the hot face of the thermoelectric cell. The investigation indicates that for a small amount of electric power input, a tangible amount of water can be collected and at the same time, dehumidify the incoming air.

Key words: Thermoelectric couple, saturation humidity ratio, electric power input.

Introduction

A vast range of thermoelectric materials have been investigated for a wide range of applications [Ref. 1]. As a result of their versatility and simplicity, many researchers have delved into optimizing their efficiencies [refs. 2, 3 and 4].

A simple Thermo-Electric heat pump[Fig. 1] is taken into consideration to dehumidify moist air and at the same time produce water. This technique is most valuable in places where hot and muggy/humid weather is prevalent. The counter flow heat exchanger[Fig. 3] is

represented by two counter flow channels whereby warm air pumped in on one side and cooled air on the other side. In the middle of the channel sits the Thermo-Electric Couple whose cold face with temperature, T_{SC} , is exposed to the ambient and humidity ratio of T_i and ω_i , respectively. The cold surface temperature, T_{SC} is kept lower than the incoming ambient temperature, T_i , so that condensation takes place to produce water (refs. 7 and 8). An electric power input is provided to the thermo-electric cell [Fig. 2] so that the cold side heat, \dot{Q}_L , which has temperature, T_{SC} , is pumped to the hot side of the thermo-Electric element with heat, \dot{Q}_H , and of temperature, T_{SH} .

In order to study the viability of Thermo-Electric heat pump for this purpose, optimization and analysis is carried out using different temperature scales. For the counter flow heat exchanger [Fig. 3], the first and second laws of Thermodynamics analysis are used. Thereafter, an analytical closed form expressions of the temperature distributions and humidity or vapor equations are derived. This process is simplified by using the analogy between heat and mass transfer on the wet/cold side of the Thermo-electric element. Since the cold side temperature of the element is lower than the incoming dew point temperature of the air, condensation of water vapor takes place on the cold side of the element. Also, the saturation humidity ratio of the cold side, ω_{SC} , is kept lower than the incoming air humidity, ω_i . As a consequence of this humidity difference, mass transfer occurs from the incoming air to the cold side of the thermo-Electric element [Refs. 5 and 6].

Determination of condensed liquid water on the cold side of the element:

The rate at which the liquid mass on the cold side condenses is given by:

$$d\dot{m}_l = \rho h_D (\omega - \omega_{SC}) dA \quad (1)$$

If a unit depth of the thermo couple is considered, the area, dA , will be expressed as $1 * dx = dx$:

$$d\dot{m}_l = \rho h_D (\omega - \omega_{SC}) dx \quad (2)$$

The amount of air condensed on the wet side is equal to the amount of vapor extracted from the incoming/ambient humid air. This is expressed as:

$$d \dot{m}_i = -\dot{m}_{air_humid} d\omega \quad (3)$$

Equating equations (2) and (3) are rearranged to yield:

$$\frac{d\omega}{dx} = -\frac{\rho h_D}{\dot{m}_{air_humid}} [\omega - \omega_{sc}] \quad (4)$$

For this analysis, we are assuming that the flow is laminar with a Reynolds number equal to:

$$Re_L = \frac{\rho u_\infty L}{\mu} = \frac{u_\infty L}{\nu} = 5 * 10^5 \quad (5)$$

The analogy or equivalence between the mass and heat transfers are expressed in terms of the Nusselt and Sherwood numbers. The Nusselt, Nu_x , and Sherwood, Sh_x , numbers describe the heat and mass transfer respectively and are given as follows:

$$Nu_x = \frac{hx}{k} = 0.332 * Re_x^{\frac{1}{2}} * Pr^{\frac{1}{3}} \quad (6)$$

and

$$Sh_x = \frac{h_D x}{D_{va}} = 0.332 * Re_x^{\frac{1}{2}} * Sc^{\frac{1}{3}} \quad (7)$$

Where the Prandtl and Schmidt numbers are defined as:

$$Sc_{T=25^0 C} = \frac{\mu}{\rho D_{va}} \approx 0.68 \quad (8)$$

Since the Prandtl and Schmidt number are very close to each other, then the Nusselt and Sherwood numbers are approximately the same. Hence, the analogy that the heat and mass transfers are equivalent is justified.

From equation (7), the expression for the coefficient for the mass transfer is:

$$h_D = 0.332 * Re_x^{\frac{1}{2}} * Sc^{\frac{1}{3}} * \frac{D_{va}}{x} \quad (9)$$

Inserting equation (9) into equation (4) and integrating the resulting equation yields:

$$\ln \left[\frac{\omega - \omega_{SC}}{\omega_i - \omega_{SC}} \right] = \frac{-0.664 * \rho_{air_humid} * D_{va} Sc^{\frac{1}{3}}}{\dot{m}_{air_humid}} \sqrt{\frac{u_{\infty} x}{\nu}}$$

(10)

For the free stream velocity in equation (10), we can compute the air flow speed from the Reynolds number as:

$$u_{\infty} = \frac{\nu * Re_L}{L} [m / sec]$$

(11)

and the mass flow is:

$$\dot{m}_{air} = \rho_{air_humid} u_{\infty} A = \rho_{air_humid} u_{\infty} wL [kg / sec]$$

(12)

Let us define:
$$a = \frac{0.664 * \rho * D_{va} Sc^{\frac{1}{3}}}{\dot{m}_{air_humid}} \sqrt{\frac{u_{\infty}}{\nu}}$$
 (13)

Then equation (10) is becomes:

$$\frac{\omega - \omega_{SC}}{\omega_i - \omega_{SC}} = e^{-a\sqrt{x}}$$

(14)

The humidity ratio at the exit (x=L) can be expressed as:

$$\frac{\omega_L - \omega_{SC}}{\omega_i - \omega_{SC}} = e^{-a\sqrt{L}}$$

(15)

From equation (15), we can derive the relationship between the ambient and exit humidity quantities as:

$$\omega_i - \omega_L = (\omega_i - \omega_{SC}) \left[1 - e^{-a\sqrt{L}} \right]$$

(16)

From Equation (3), the mass of condensed liquid is equivalent to the mass of water vapor lost by the air due to condensation. Then, integrating equation (3) for the over length, L, of the Thermo-Electric cell and using equation (14), the liquid mass is:

$$\dot{m}_l = -\dot{m}_{air_humid} (\omega_L - \omega_i) = \dot{m}_{air_humid} (\omega_i - \omega_L)$$

(17)

Determination of the temperatures on Cold and hot sides of the Thermo-Electric element:

The sensible heat transferred from the cold side is equal to the heat conducted across the channel to the hot side of the thermo-Element.

$$d\dot{q} = Ah(T - T_{SC}) = \dot{m}_{air_humid} c_p dT$$

(18)

Using the expression for h from the Nusselt number:

$$h = 0.332 * \frac{k * Re_x^{\frac{1}{2}} * Pr^{\frac{1}{3}}}{x}$$

(19)

$$\frac{dT}{T - T_{SC}} = 0.332 * \frac{k * Pr^{\frac{1}{3}}}{x} \sqrt{\frac{u_\infty x}{\nu}}$$

(20)

After integrating it and letting:

$$b = \frac{-0.664 * k * Pr^{\frac{1}{3}} * \sqrt{\frac{u_\infty}{\nu}}}{\dot{m}_{air} c_p}$$

(21)

Then the temperature distribution (equation (20)) on the cold face of the couple becomes:

$$\frac{T_{x=L} - T_{SC}}{T_i - T_{SC}} = e^{-b\sqrt{L}} \Rightarrow$$

or

$$(T_{x=L} - T_i)_{cold_face} = [T_i - T_{SC}] [e^{-b\sqrt{L}} - 1]$$

(22)

In the same manner, we can show that the temperature distribution on the hot face is:

$$\frac{T_{x=L} - T_{SH}}{T_i - T_{SH}} = e^{b\sqrt{L}} \Rightarrow$$

or

$$(T_{x=L} - T_i)_{hot_face} = [T_{SH} - T_i] [1 - e^{b\sqrt{L}}]$$

(23)

It is also known that the amount heat absorbed by the cold face and respectively the heat extracted from the hot face of the thermoelectric unit are:

$$\dot{Q}_L = \dot{m}_a c_p (T_{x=L} - T_i)_{cold_face} + \dot{m}_a (\omega_i - \omega_L) h_{fg}$$

(24)

$$\dot{Q}_H = \dot{m}_a c_p (T_{x=L} - T_i)_{hot_face}$$

(25)

We insert equation (22) into equations (17) and (24), respectively, to compute the amount of condensed water and absorbed heat on the cold face of the thermoelectric couple. In the same manner, equation (23) is inserted into Equation (25) to compute the rate of heat extracted from the hot face of the thermo couple.

The performance data of the thermoelectric pump [Fig. 1] is provided by the manufacturer [Ref.7] on [Fig. 2] as:

$$\dot{Q}_H = A + B * (T_{SH} - T_{SC})$$

(26)

$$\dot{Q}_L = C + D * (T_{SH} - T_{SC})$$

(27)

where A, B, C and D are constants that are determined from the removed and absorbed heat graphs [Fig. 2].

Numerical Optimization Procedure:

As provided by equations (26) and (27), the salient characteristics of the thermoelectric module [Fig. 1] were computed by linear interpolation from the graphs [fig. 2]. Also the heat pump is considered to be one meter long and 0.02 meter deep. To initiate the numerical optimization, the incoming ambient temperature and humidity values are predicted. One must be cognizant that, for the condensation to take place on the cold face, the ambient temperature and humidity need to be higher than the temperature and humidity on the cold

surface. Since the temperatures on the hot and cold sides of the thermo couple are unknown apriori, iterative method is adopted as follows:

- a). Guess an initial value for the temperature gradient: $\Delta T = T_{SH} - T_{SC}$
- b). Use equations (26) and (27) to compute the new values of \dot{Q}_H and \dot{Q}_L .
- c). Use the new found values of \dot{Q}_H and \dot{Q}_L into equations (22) and (23) to determine the updated values of T_{SH} and T_{SC} .
- d). Compute the new $\Delta T = T_{SH} - T_{SC}$
- e). Check if the convergence is met, i.e., if $\Delta T \leq \varepsilon$, where epsilon is set to a very small value.
- f). Once convergence is achieved, compute the liquid condensation rate by using equation (17).

Numerical optimization results:

Using the converged values, the liquid mass rate (eq. 17) is computed to be 2.532 kg/hour. Along with the condensed mass of liquid condensed, the converged values of humidity and temperatures are shown on Table 1.

The analysis indicates that once the condensation is complete and the heat is removed from the hot face of the couple, the temperature on the hot side subsides from 87.8035 degrees to 48.9629 degrees Celsius.

In graph 1, the converged solution of parameters at every grid point of the thermo couple length is presented. It should be remembered that these values are used to compute the condensed liquid over the cold surface of the thermoelectric element as presented on Table 1. As presented on Table 2, graphs 4 and 5, the temperatures on both sides of the thermoelectric faces converge to the dew-point and ambient temperatures. This is achieved by the successfully dehumidifying the incoming air and thereby extracting the heat and producing water by the thermoelectric couple. The extracted and absorbed heat rates are presented on Fig. 5. The heat removed and water condensed can be enhanced by putting stacks of thermoelectric elements and various external physical mechanisms such as fins.

Conclusion:

A thermoelectric device was used to serve dual purposes, namely to cool the incoming air and produce water. In doing so, the air is dehumidified resulting in comfort that can be useful in places where humidity is extremely high. To increase the efficiency of the thermocouple, we

intend to incorporate various sizes of fins and incorporate Computational Fluid Dynamics to fully analyze the flow field.

$\dot{m}_{liquid} \left[\frac{kg}{hr} \right]$	$\omega_{x=L} \left[\frac{kg_{H_2O}}{kg_{dry_air}} \right]$	$T_{SH} [^{\circ}C]$	$[^{\circ}C] T_{SC}$	$T_{x=L} [^{\circ}C]$
2.53152	0.056222	48.9629	33.5555	42.7058

Table 1: Optimum values at x=L.

X[m]	TSH[Celsius]	TSC[Celsius]	Q_dot_H[kwatts]	Q_dot_L[kwatts]
0.005	85.6599	37.3002	0.0367654	0.00617589
0.1	60.139	38.5276	0.0658227	0.0306745
0.2	55.5834	36.5921	0.0681536	0.0326397
0.3	53.4543	35.642	0.0691974	0.0335198
0.4	52.1523	35.0482	0.0698233	0.0340474
0.5	51.2497	34.6311	0.070252	0.0344089
0.6	50.576	34.317	0.0705693	0.0346764
0.7	50.0481	34.0692	0.0708164	0.0348847
0.8	49.6198	33.8672	0.0710159	0.0350529
0.9	49.2632	33.6982	0.0711813	0.0351924
0.999	48.9629	33.5555	0.0713201	0.0353094

Table 2: Temperature and heat distributions along the cold and hot faces of the thermo-couple.

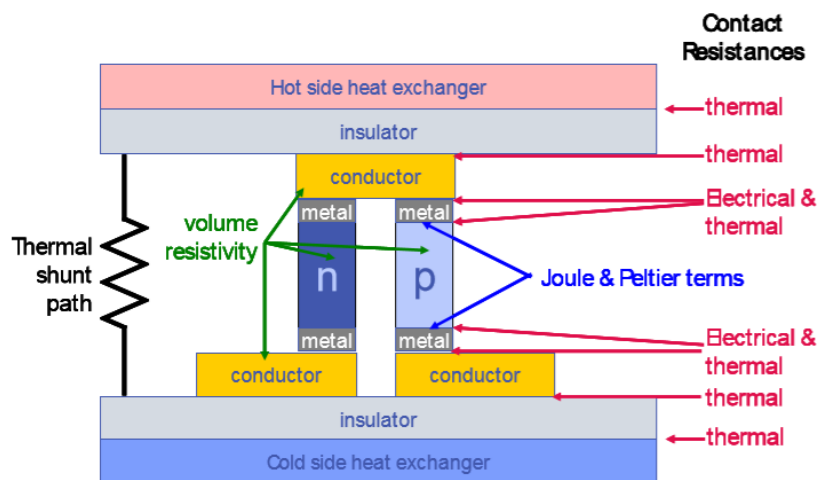


Fig. 1: General Feature of Thermoelectric Module (www.tetech.com)

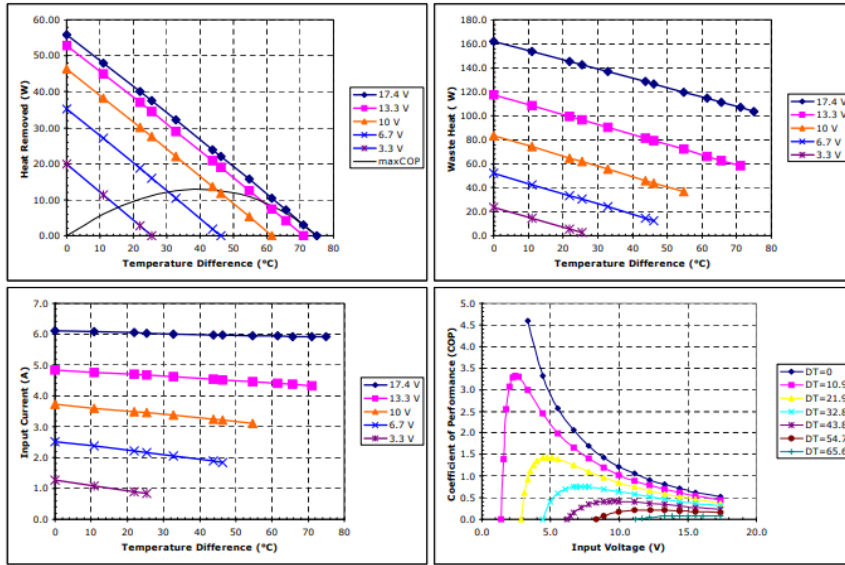


Fig. 2. Peltier-Thermoelectric Module chart (www.tetech.com)

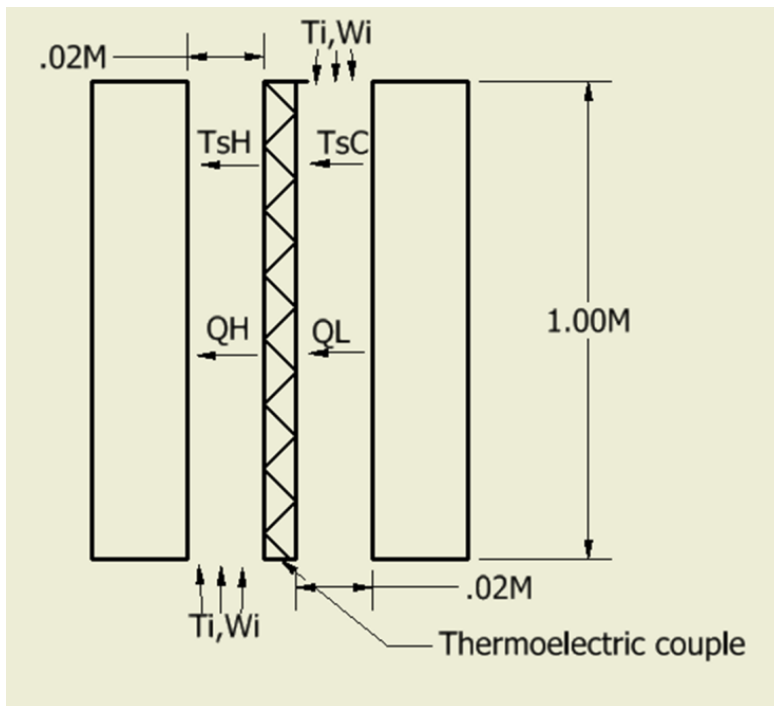


Fig. 3. Counter flow mechanism in Thermo-electric couple.

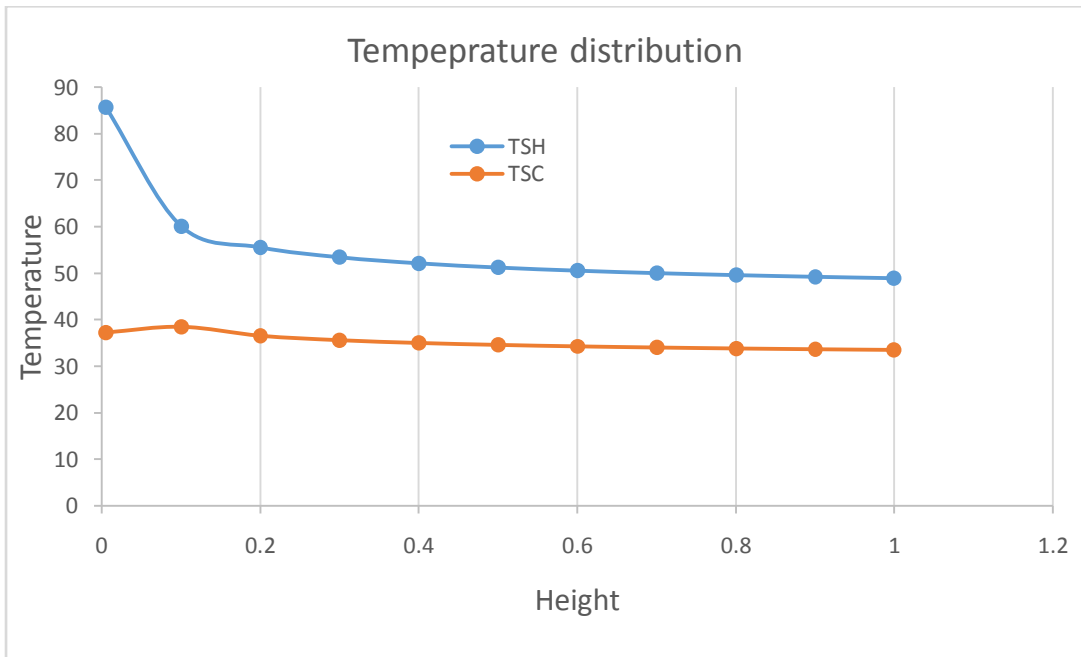


Fig. 4: Convergence history of temperature.

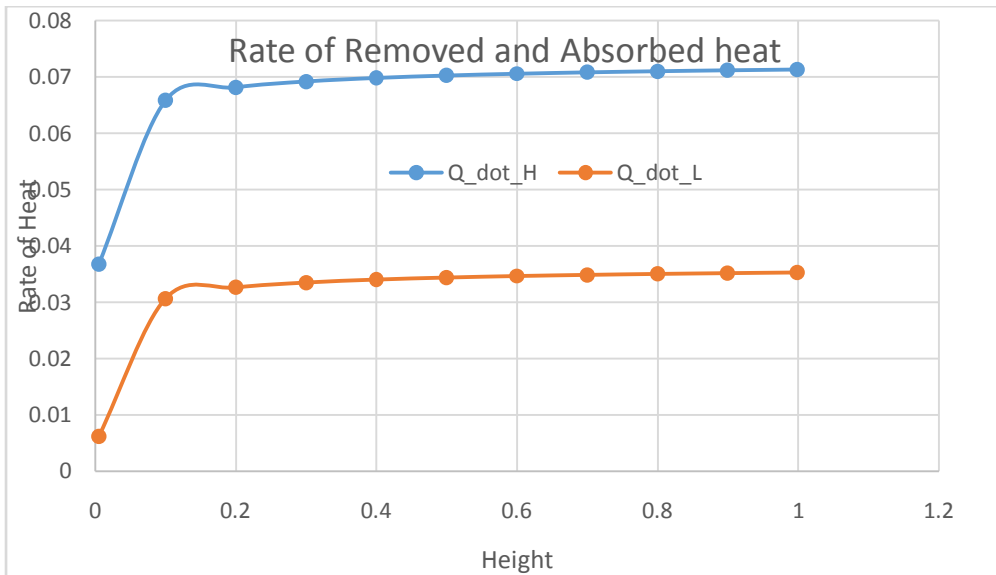


Fig. 5Rate of heat convergence

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