International Research Journal of Mathematics, Engineering and IT
Vol. 3, Issue 1, Jan 2016 IF- 2.868 ISSN: (2349-0322)

Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

HOMOMORPHISM AND CARTESIAN PRODUCT OF ANTI Q-FUZZY BG-IDEALS IN BG-ALGERBA

K. Jaibunisha
M.Phil. Research Scholar, Department of Mathematics,
Jamal Mohamed College, Tiruchirappalli – 20.
A. Prasanna
Assistant Professor, Department of Mathematics, Jamal Mohamed College,
Tiruchirappalli – 20.
S.Ismail Mohideen
Associate Professor, Department of Mathematics, Jamal Mohamed College,
Tiruchirappalli – 20.
A. **Solairaju**
Associate Professor, Department of Mathematics, Jamal Mohamed College,

Tiruchirappalli – 20

ABSTRACT

In this paper, we introduce the notion of Anti Q- fuzzy BG- ideal in BG-Algebra under homomorphism and Cartesian product, lower level cut of a fuzzy set, lower level BG-ideal and prove some result on these. We show that a Q-fuzzy subset of a BG-algebra is a Q-fuzzy BG-ideal if and only if complement of this Q-fuzzy subset is an anti Q-fuzzy BG-ideal.

Keywords

BG-algebra, sub BG-algebra, BG-ideal, fuzzy BG-ideal, Anti fuzzy BG-ideal, Q-fuzzy BG-ideal, Anti Q-fuzzy BG-ideal, Homomorphism and Cartesian product.

1.Introduction

The concept of fuzzy set was introduced by Zadeh [8].Since then these ideas have been applied to other algebric structure such as semigroups, groups rings, models vector spaces and topologies. J,.Neggers [6] and H.S.Kim[6] introduced the new notion, called B-algebra. R.Biswas [1] introduced the concept of anti fuzzy subgroup of groups. Modifying his idea, in this paper, we apply the idea to BG-algebra. T.Priya [9] and T.Ramachandran [9] discussed the concept of Homomorphism and Cartesian product of Fuzzy PS-algebras. We introduce a notion Homomorphism and Cartesian product of Anti Q-fuzzy BG-ideals in BG-algebras, Lower level cuts of a Q-fuzzy set, and prove some result on these. In this paper, we classify the Homomorphism and Cartesian product of Anti Q-fuzzy BG-ideal in BG-algebra.

2.Preliminaries

In this section we site the fundamental definitions that will be used in the sequel.

Definition 2.1

A non empty set X with a constant 0 and a binary operation '.' is called a BG-Algebra if it satisfies the following axioms.

1. $x * x = 0$,
2. $0 * x = x$,
3. $(x * y) * (y * x) = x$, for all $x, y \in X$.

Example 2.1

Let = {0, a, b} be the set with the following table

*	0	a	b
0	0	a	b
a	a	0	a
b	b	b	0

Then (X, $*$, 0) is a BG-Algebra.

We can define a relation (partial ordering) $x \leq y$ if and only if $x * y = 0$.

Preposition 2.1

In any BG-algebra X , the following hold:

1. $x * y \leq 0$
2. $(x * y) * (x * y) \leq x * y$
3. $x * (x * (x * y) = x * y$
4. $x \leq y$ implies $x * z \leq y * z$ and $z * y \leq z * x$

Definition 2.2

Let S be a non-empty subset of a BG-algebra X, then S is called a

sub algebra of X if $x * y \in S$, for all $x, y \in S$.

Definition 2.3

Let X be a BG –algebra and I be a subset of X, then I is called a BG- right ideal of X if it satisfies the following conditions:

1. $0 \in I$,
2. $x * y \in I$ and $y \in I \Rightarrow x \in I$,
3. $x \in I$ and $y \in X \Rightarrow x * y \in I$, $I \times X \subseteq I$.

Definition 2.4

Let X be a BG –algebra and I be a subset of X, then I is called a BG- left ideal of X if it satisfies the following conditions:

1. $0 \in I$,
2. $x * y \in I$ and $y \in I \Rightarrow x \in I$,
3. $y \in I$ and $x \in X \Rightarrow y * x \in I$, $I \times X \subseteq I$.

Definition 2.5

Let X be a BG –algebra and I be a subset of X, then I is called a BG- ideal of X if it satisfies the following conditions:

1. $0 \in I$,
2. $x * y \in I$ and $y \in I \Rightarrow x \in I$,
3. $x \in I$ and $y \in X \Rightarrow x * y \in I$, and $y * x \in I$, $I \times X \subseteq I$.

Definition 2.6

Let X be a non-empty set. A fuzzy set α of the set X is a mapping α: X→[0,1].

Definition 2.6

Let Q and G be any two sets. A mapping A: $G \times Q \rightarrow [0,1]$, is called a Q- fuzzy set in G.

Definition 2.7

Let αbe a Q-fuzzy set in set X. For $t \in [0,1]$, the set $\alpha_t = \{x \in X \,/\, \alpha(x,q) \geq t$ for all $q \in Q\}$ is called level fuzzy subset of α.

Definition 2.8

If α be a Q-fuzzy set in X. Then the complement denoted by α^c is the Q-fuzzy subset of X given by α^c(x, q)= 1- α(x, q),for all x ∈ X and q ∈ Q.

Definition 2.9

Let α be a Q- fuzzy BG-algebra. Then α is called Q-fuzzy sub algebra of x if $\alpha(x * y, q) \geq \min\{\alpha(x, q), \alpha(y, q)\}$, for all x,y ∈ X and q ∈ Q.

Definition 2.10

A Q-fuzzy set α in X is called Q-fuzzy BG- ideal of X if it satisfies the following the following inequality, For all x, y ∈ X and q ∈ Q,

1. $\alpha(0, q) \geq \alpha(x, q)$,
2. $\alpha(x, q) \geq \min\{\alpha(x * y, q), \alpha(y, q)\}$,
3. $\alpha(x * y, q) \geq \min\{\alpha(x, q), \alpha(y, q)\}$.

Homomorphism of anti Q-fuzzy ideals

In this section we discuss about anti Q-fuzzy BG-ideals and BG- algebra under homomorphism and some of their properties

Definition 3.1

A Q- fuzzy set α BG-algebra X is called anti Q-fuzzy sub algebra of X if

$\alpha(x * y, q) \leq \max\{\alpha(x, q), \alpha(y, q)\}$, for all x, y ∈ X and q ∈ Q.

Definition 3.2

A Q-fuzzy set α of BG-algebra X is called an anti Q-fuzzy BG- ideal of X if for all x,y ∈ X and q ∈ Q,

1. $\alpha(0, q) \leq \alpha(x, q)$,
2. $\alpha(x, q) \leq \max\{\alpha(x * y, q), \alpha(y, q)\}$,
3. $\alpha(x * y, q) \leq \max\{\alpha(x, q), \alpha(y, q)\}$.

Example 3.1

Let X={0,a,b,c} be the set with the following table.

$*$	0	a	b	c
0	0	a	b	c
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

Let t_o, t_1, $t_2 \in [0,1]$ be such that $t_o < t_1 < t_2$. Define a Q-fuzzy set $\alpha : X \times Q \to [0,1]$ by $\alpha(0,q) = t_o$, $\alpha(a,q) = t_1 = \alpha(b,q)$ and $\alpha(c,q) = t_2$, routine calculation α is an anti Q-fuzzy subalgebra of X, and it is an anti fuzzy BG-ideal of X and $q \in Q$.

Definition 3.3

Let (X, $*$, 0) and (Y, $*$, 0) be BG-algebras. A mapping f: X $\to$Y is said to be homomorphism $f(x * y) = f(x) * f(y)$, for all x, y $\in$ X.

Remark

If f : x $\to$ y is a homomorphism of BG-algebra then f(0) = 0.

Definition 3.4

Let f : X $\to$ Y be an endomorphism and α be a fuzzy set in X.We define a new fuzzy set in X. α_f in X as $\alpha_f(x) = \alpha(f(x))$ for all x in X.

Theorem 3.1

Let f: X $\to$ Y be endomorphism of BG-algebra, Let α is an anti Q-fuzzy BG-ideal of X if and only if α_f is anti Q-fuzzy sub algebra of X.

Proof

By definition, Every anti Q-fuzzy BG-ideal of a BG-algebra X is an anti Q-fuzzy sub algebra of X.

Conversely, let αbe an anti Q-fuzzy subalgebra of X.

To prove: α_f is an anti Q-fuzzy subalgebra of X. For all x,y $\in$ X and q$\in$Q,

$\alpha_f(0) = \alpha(f(0))$

$\leq \alpha(f(x))$

$= \alpha_f(x)$ for all $x \in X$.

$\alpha_f(x,q) = \alpha(f(x * y) * (0 * y), q)$

$\leq \max \{(\alpha(f(x * y, q), \alpha(f(0 * y, q))$

$\leq \max \{(\alpha(f(x * y, q), \max \{\alpha(f(0, q) * f(y, q)\}\}$

$\leq \max \{(\alpha(f(x * y, q), \alpha(f(y, q))\}$

$= \max \{(\alpha_f(x * y, q), \alpha_f(y, q))\}$

(ie) $\alpha_f(x,q) \leq \max \{(\alpha_f(x * y,q), \alpha_f(y,q))\}$

Hence α_f is an anti Q-fuzzy BG-ideal of X.

Theorem 3.2

Let $f : X \rightarrow Y$ be an endomorphism of BG-algebra. Let α be anti Q-fuzzy BG-ideal of a BG-algebra X. If the inequality $x * y \leq z$ holds in X. Then

$\alpha_f(x, q) \leq \max\{ (\alpha_f(y, q), \alpha_f(z, q))\}$ for all $x, y, z \in X$ and $q \in Q$.

Proof

Assume the inequality $x * y \leq z$ hold in X, and $\alpha$is an anti Q-fuzzy BG-ideal of X.

Now,

$\alpha(0) = \alpha(f(0)) = \alpha_f(0) \leq \alpha_f(x) \ \alpha(f(x))$

$\alpha_f(x* y, q) \leq \alpha(f(x * y), q)$

$\leq \max\{(\alpha(f(x * y * z, q), \alpha(f(z, q))\}$

$= \max\{(\alpha(f(0, q),\alpha(f(z, q))\}$

$= \alpha(f(z, q))$

$= \alpha_f(z, q)$

It follows that,

$$\alpha_f(\text{x, q}) \leq \alpha(\text{f(x, q)})$$

$$\alpha_f(\text{x, q}) \leq \max\{ (\alpha(\text{f(x} * \text{y, q)}, \alpha(\text{f(y, q))}\}$$

$$\leq \max\{ (\alpha(\text{f(z, q)}, \alpha(\text{f(y, q))}\}$$

$$\leq \max\{\alpha_f(\text{z, q}) , \alpha_f(\text{y, q})\}$$

Hence the result.

Theorem 3.3

Let f : X →Y is homomorphism of BG-algebra.A Q-fuzzy subset α_f of a BG-algebra X is a Q-fuzzy BG-ideal of X if and only if its complement α_f^c is an anti Q-fuzzy BG-ideal of X.

Proof

Let α_f be an anti Q-fuzzy BG-ideal of X and let x,y ∈ X and q ∈ Q.

Then,

(i) $\alpha_f^c(\text{0, q}) = \alpha^c(\text{f(0, q)})$

$$= 1 - \alpha(\text{f(0, q)})$$

$$\leq 1 - \alpha(\text{f(x, q)})$$

$$\leq 1 - \alpha_f(\text{x, q})$$

$$= \alpha_f^c(\text{x, q})$$

(ii) $\alpha_f^c(\text{x, q}) = \alpha^c(\text{f(x, q)})$

$$= 1 - \alpha(\text{f(x, q)})$$

$$\leq 1 - \min\{\alpha(\text{f(x} * \text{y, q)}\alpha(\text{f(y, q)}\}$$

$$= 1 - \min\{ 1 - \alpha^c(\text{f(x} * \text{y, q)} , 1 - \alpha^c(\text{f(y, q)}\}$$

$$= \max\{\alpha^c(\text{f(x} * \text{y, q)}, \alpha^c(\text{f(y, q)}\}$$

That is

$$\alpha_f^c(\text{x, q}) \leq \max \{\alpha_f^c(\text{x} * \text{y, q}), \alpha_f^c(\text{x, q})\}$$

(iii) $\alpha_f^c(x * y, q) = \alpha^c(f(x* y, q))$

$= 1-\alpha(f(x * y, q))$

$\leq 1-\min\{\alpha(f(x, q)\ f(y, q))\}$

$= 1-\min\{\alpha(f(x, q)\ , \alpha(f(y, q))\}$

$= 1-\min\{ 1-\alpha^c(f(x, q)\ ,1-\alpha^c(f(y, q)\}$

$= \max\{\alpha^c(f(x, q),\alpha^c(f(y, q)\}$

$\leq \max\ \{\alpha_f^c(x, q),\alpha_f^c(y, q)\}$

That is,

$\alpha_f^c(x* y, q) \leq \max\ \{\alpha_f^c(x, q),\alpha_f^c(y, q)\}$

Thus α_f^c is an anti Q-fuzzy ideal of X the converse also can be proved similarly.

Cartesian product of anti Q-fuzzy BG-ideal in BG-algebra

In this section, we introduce the concept of Cartesian product of anti Q-fuzzy BG-ideal in BG-algebra.

Definition 4.1

Let α be a Q-fuzzy subset of a BG-algebra X. For t ∈ [0,1], the set

$\alpha^t = \{x \in X\ \ \alpha(x,q) \leq t\}$ is called a lower cut of α cleary and $\alpha^1 = X$ and

$\alpha_t \cup \alpha^t = X$ for t ∈ [0,1]. If $t_1 < t_2$ then$\alpha^{t1} \subseteq \alpha^{t2}$.

Definition 4.2

Let α and δ be the fuzzy sets in X. The Cartesian product

$(\alpha \times \delta)$: $X \times X \rightarrow [0,1]$ is defined by $(\alpha \times \delta)\ (x, y) = \min\ \{\alpha(x),\delta(y)\}$ for all

x, y ∈ X.

Definition 4.3

Let α and δ be the anti Q-fuzzy BG-ideal in X. The Cartesian Product

$(\alpha \times \delta)$: $X \times X \rightarrow [0,1]$ is defined by $(\alpha \times \delta)\ (x,y) = \max\ \{\alpha(x),\ \delta(y)\}$ for all x, y ∈ X.

Theorem 4.1

Let α and δ be an anti Q-fuzzy subset of a BG-algebra X × X. If $\alpha \times \delta$ is anti Q-fuzzy BG-ideal of X × X. Then the Lower level cut α^t is a BG-ideal of X for all t ∈ [0,1]; t ≥ α(0, q).

Proof

Let α and δ be an anti Q-fuzzy BG-ideal of X × X. Then for all

x,y ∈ X × X and q ∈ Q.

$$(\alpha \times \delta)\,(0, 0) \leq \max\{\alpha(0), \delta(0)\}$$

$$\leq \max\{\alpha(x)\,,\, \delta(y)\}$$

$$\leq (\alpha \times \delta)\,(x, y)$$

If α is an anti Q-fuzzy ideal of X. Then for all x, y ∈ X and q ∈ Q.

1. $\alpha(0, q) \leq \alpha(x, q)$,
2. $\alpha(x, q) \leq \max\{\alpha(x * y, q),\ \alpha(y, q)\}$,
3. $\alpha(x * y, q) \leq \max\{\alpha(x, q),\ \alpha(y, q)\}$.

To prove that α^t is an BG-ideal of X.

We know that $\alpha^t = \{x \in X \,/\, \mu(x, q) \leq t\}$

Let x, y ∈ α^t and α is an anti Q-fuzzy BG-ideal of X .

Since $\alpha(0,q) \leq \alpha(x,q) \leq t$ implies $0 \in \alpha^t$,for all t∈[0,1]

Let x∗y ∈ α^t and y ∈ α^t

Therefore $\alpha(x * y, q) \leq t$ and $\alpha(y, q) \leq t$

Now, $\alpha(x,q) \leq (\alpha \times \delta)\,((x, 0), q)$

$$\leq \max\{(\alpha \times \delta)\,((x, 0)* (y, 0), q),\ (\alpha \times \delta)\,((y, 0),q)\}$$

$$\leq \max\{(\alpha \times \delta)\,((x * y), (0 * 0), q), (\alpha \times \delta)\,((y, 0), q)\}$$

$$\leq \max\{(\alpha \times \delta)\,((x * y, 0), q), (\alpha \times \delta)\,((y,0), q)\}$$

$$\leq \max\{\alpha(x * y, q), \alpha\,(y, q)$$

$$\leq \max\{t,t\}$$

$\leq$ t

Hence α(x, q) $\leq$ t

That is x $*$ y $\in \alpha^t$ and y $\in \alpha^t$ Implies x $\in \alpha^t$.

(i) Let x $\in \alpha^t$ and y $\in$ X

Choose y in X such that, α(y, q) $\leq$ t

Since x $\in \alpha^t$ implies α(x, q) $\leq$ t

We know that

$$
\begin{aligned}
\alpha(x * y, q) &= (\alpha \times \delta)\ ((x * y, 0), q) \\
&= (\alpha \times \delta)\ ((x * y), (0 * 0), q) \\
&= (\alpha \times \delta)\ ((x, 0), *(y, 0), q) \\
&\leq \max\{(\alpha \times \delta)\ ((x, 0), q), (\alpha \times \delta)\ ((y, 0), q)\} \\
&\leq \max\{\alpha(x, q), \alpha\ (y, q)\} \\
&\leq \max\{t, t\} \\
&\leq t.
\end{aligned}
$$

That is, α(x $*$ y, q) $\leq$ t implies x $*$ y $\in \alpha^t$

(ii) Let y $\in \alpha^t$, x $\in$ X

Choose x in X such that ,α(x, q) $\leq$ t

Since y $\in \alpha^t$ implies α(y, q) $\leq$ t

We know that

$$
\begin{aligned}
\alpha(y * x, q) &= (\alpha \times \delta)\ ((y * x, 0), q) \\
&= (\alpha \times \delta)\ ((y * x), (0 * 0), q) \\
&= (\alpha \times \delta)\ ((y, 0), *\ (x, 0), q) \\
&\leq \max\{(\alpha \times \delta)\ ((y, 0), q), (\alpha \times \delta)\ ((x, 0), q)\} \\
&\leq \max\{\alpha(y, q), \alpha\ (x, q)\} \\
&\leq \max\{t, t\}
\end{aligned}
$$

$\leq$ t

That is, α(y $*$ x, q) $\leq$ t implies y $*$ x $\in \alpha^t$

Hence α^t is a BG- ideal of X.

Theorem 4.2

Let α and δ be a Q-fuzzy subset of a BG- algebra of X, such that $\alpha \times \delta$ is an anti Q-fuzzy ideal of X $\times$ X. If for each t $\in$ [0,1],t $\geq \alpha$(0, q).The lower level cut $\alpha^t$is a BG ideal of X. Then α is an anti Q-fuzzy BG-ideal of X.

Proof

Since $\alpha^t$is a BG ideal ofX.

(i) 0 $\in \alpha^t$

(ii) x $*$ y $\in \alpha^t$ and y $\in \alpha^t$implies x $\in \alpha^t$

(iii) x $\in \alpha^t$ and y $\in$ X implies x $*$ y $\in \alpha^t$.

To prove that $\alpha^t$is an anti Q-fuzzy BG-ideal of X.

For all x, y $\in$ X and q $\in$ Q.

(i) Let x, y $\in \alpha^t$ then α (x, q) $\leq$ t, α (y, q) $\leq$ t

Let α(x, q) $\leq t_1$ and α(y, q) $\leq t_2$,

Without loss of generality let $t_1 \leq t_2$,

Then x $\in \alpha^{t_2}$

Now x $\in \alpha^{t_2}$ and y $\in$ X. x $*$ y $\in \alpha^{t_2}$.

That is,

α(x $*$ y, q) $\leq t_2$

= max $\{t_1, t_2\}$

= max{max $\{t_1, t_2\}$,max $\{t_1, t_2\}$}

= max {max{ α(x, q) α(y, q), max{δ(0, q) δ(0, q)}}

= max{max{α(x, q) α(y, q)} ,δ(0, q)}

= max {($\alpha \times \delta$) ((x,0), q), ($\alpha \times \delta$) ((y,0), q)}

$= \max\{ \alpha(x, q)\, \alpha(y, q)\}$

(ii) Let $\alpha(0, q) = \alpha(x * x, q)$

$= (\alpha \times \delta)((x * x, 0), q)$

$= (\alpha \times \delta)((x * x, 0 * 0), q)$

$= (\alpha \times \delta)\ ((x, 0) *(x, 0), q)$

$\leq \max\{(\alpha \times \delta)\ ((x, 0), q), (\alpha \times \delta)\ ((x, 0), q)$

$\leq \max\ \{\alpha(x, q), \alpha(x, q)\}$

$\leq \alpha\ (x, q)$

Therefore $\alpha(0, q) \leq \alpha\ (x, q)$

(iii)Let $\alpha(x, q) = (\alpha \times \delta)((x, 0), q)$

$= (\alpha \times \delta)((x, 0) * (y, 0)) * ((0, 0) * (y, 0)), q)\}$

$\leq \max\{ (\alpha \times \delta)((x, 0) * (y, 0), q),$

$(\alpha \times \delta)\ ((0, 0) *(y, 0), q\}$ by(i)

$\leq \max\{ (\alpha \times \delta)((x, 0) * (y, 0), q),$

$\max\ \{(\alpha \times \delta)\ ((0, 0), q), (\alpha \times \delta)\ (y, 0)), q\}$

$\leq \max\{ (\alpha \times \delta)((x * y),(0 * 0), q),(\alpha \times \delta)\ (y, 0)), q\}$

$\leq \max\ \{ \alpha(x * y, q), \alpha(y, q) \},$ by(ii)

Therefore $\alpha(x, q) \leq \max\ \{ \alpha(x * y, q), \alpha(y, q) \},$

Hence α is an anti Q-fuzzy BG-ideal of X.

5.Conclusion

In this paper we have discussed anti Q-fuzzy BG-ideal and BG-sub algebras of BG-algebra under homomorphism and Cartesian product. It has observed that BG-algebras as a generalization of BCK/BCI/B/d-algebras.

REFERENCES

[1] Biwas.R , Anti fuzzy sub groups of groups,9March, 1990 Vol 35(1), 121-124.

[2] Jun.Y.B, Roh.E.H,andKim.H.S, On BH-algebras, sci.mathematics, 1 (1998), 347-354.

[3] Muthuraj.R ,Sridharan.M, Sitharselvam.M.P, Fuzzy BG-ideal in BG-Algebra, International Journal of Computer Application. (0975-8887) Volume 2,No.1, 26-30, May 2010.

[4] Muthuraj.R, Sitharselvam.M.P, Muthuraman.M.S, Anti Q-Fuzzy groups and its lower level sub groups, International Journal of Computer Applications (0975-8887) Volume 3, No.3, 16-20, June 2010.

[5] Muthuraj.R ,Sridharan.M, Muthuraman.M.S, Sitharselvam.M.P, Anti Q-Fuzzy BG-ideal in BG-Algebra, International Journal of Computer Application. (0975-8887) Volume 4,No.11, August 2010.

[6] Neggers and Kim.H.S, On- B-Algebras,Math Vensik,54 (2002), 21-29.

[7] Neggers and Kim.H.S, On- d-Algebras,Math Slovaca,49 (1999), 19-26.

[8] Priya.T and Ramachandran.T, Homomorphism and Cartesian product of Fuzzy PS-Algebras, Applied Mathematics Science,Vol.8,2014, no.67, 3321-3330.

[9] Zadeh.L.A, fuzzyset, Inform.Comtrol,8(1965),338-353.