PHONON-PHONON INTERACTION IN RAMAN SCATTERING OF GERMANIUM

A. Chauhan and S. C. Gairola
Department of Physics(School of Sciences) H. N. B. Garhwal University
(A Central University) Campus Pauri, Pauri, Pincode-246001, Uttarakhand, India.

ABSTRACT

Germanium has diamond type structure. It has two identical atoms in primitive unit cell. An equation of motion technique of quantum dynamics has been applied to develop the theory of Raman spectra of germanium. An expression for electron phonon linewidth and electron phonon shift has been obtained. It is established fact that at high temperature limit when anharmonic effects are dominant, the contributions of harmonic field, localized field, electron electron interaction field are feeble. It has been found that at high temperature limit, cubic and quartic parts of electron phonon linewidth and electron phonon shift have been matched with the Balkanski et al. model for linewidh and shift at high temperature. An analysis of first order Raman spectra of germanium has been carried out on taking anharmonicity upto quartic terms.

KEYWORDS – Cubic anharmonic term, Equation of motion technique, Germanium, Optical phonons, Quartic anharmonic term, Raman spectra.

1. Introduction

Analysis of temperature effects on Raman silicon photonic devices[1], stimulated Raman scattering in silicon photonic crystal waveguides under continuous excitations[2], surface - and point – defect- related Raman scattering in wurtzite semiconductors excited above the band gap[3], and theory of surface-enhanced Raman scattering in semiconductors[4] are being progress in the latest technology. It is known that real crystal lattice contains host atoms in addition to impurity. This impurity destroys the symmetry of crystal. This generates localized modes [5]. In pure crystals, normal modes of vibrations are independent to one another. The quantization of these modes is known as phonon [6]. At high temperatures, the cubic and anharmonic terms are taken in potential energy expansion of crystal. This gives phonon – phonon interaction [7]. An interaction of electron with phonon creates polaron. Polaron correlation function depends on temperature and electron- phonon coupling constant [8]. According to Hall et al. electron can absorb a photon by indirect transition to conserve

the energy and momentum laws at the same time [9]. Raman scattering by phonons in a crystal is the inelastic scattering of light caused by the fluctuations in the crystal electronic polarizability induced by the displacements of the atoms from their equilibrium positions [10]. In presence of impurity, and anharmonicity, the analysis is to be done on first order Raman spectra of bulk germanium crystal by virtue of involvement of phonons of Brillouin zone centre. The work has been done on Raman scattering in anharmonic crystals [11], and semiconductor crystals [12,13] but they have not done the analysis on specific semiconductors. The formulation of the problem, evaluation of electron phonon linewidth, and electron phonon shift, analysis of first order Raman spectra of germanium, and conclusion of the work have been carried out in the following sections.

2. Formulation of the Problem

The intensity of the Raman spectrum for $\vec{k}=0$ is given as [14]

$$I \quad \propto \quad A \tag{1}$$

In eq.(1), I and A are Raman intensity and spectral density function respectively.

Eq.(1) can be written as

$$I = P \times A \tag{2}$$

In eq.(2), P represents proportionality constant. The spectral density function is given as[14]

$$A = \Gamma(0, j;\varepsilon) \Big/ \pi\left[\{\varepsilon - \varepsilon_0 - \Delta(0, j;\varepsilon)\}^2 + \Gamma^2(0, j;\varepsilon)\right] \tag{3}$$

This value of spectral density function eq. (3) gives the intensity (I) eq.(2) as

$$I = P\,\Gamma(0, j;\varepsilon) \Big/ \pi\left[\{\varepsilon - \varepsilon_0 - \Delta(0, j;\varepsilon)\}^2 + \Gamma^2(0, j;\varepsilon)\right] \tag{4}$$

In eq.(4), $\Gamma(0, j;\varepsilon)$, ε, ε_0, $\Delta(0, j;\varepsilon)$ are electron phonon linewidth for $\vec{k}=0$ optical phonon, frequency, harmonic frequency and electron phonon shift for $\vec{k}=0$ optical phonon respectively.

3. Evaluation of Electron Phonon Linewidth and Electron Phonon Shift

For the analysis of Raman spectra of germanium crystal, let us take Hamiltonian as[5,15-25]

$$H = H_{op} + H_{oe} + H_D + H_{ep} + H_A \tag{5}$$

Each parts of eq.(5) of Hamiltonian H are defined as

$$H_{op} = \left(\hbar/4\right)\sum_k \varepsilon_k \left[A_k^{*} A_k + B_k^{*} B_k\right] \tag{6a}$$

$$H_{oe} = \hbar \sum_{q} \varepsilon_q b_q^{*} b_q \qquad (6b)$$

$$H_{ep} = g\hbar \sum_{k,q} b_Q^{*} b_q B_k \qquad (6c)$$

$$H_D = \hbar \sum_{k_1,k_2} \left[D\left(\vec{k}_1, \vec{k}_2\right) A_{k_1} A_{k_2} - C\left(\vec{k}_1, \vec{k}_2\right) B_{k_1} B_{k_2} \right] \qquad (6d)$$

$$H_A = \hbar \sum_{s\geq 3} \sum_{k_1,k_2,\ldots,k_s} V^{(s)}\left(\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_s\right) A_{k_1} A_{k_2} \ldots A_{k_s} \qquad (6e)$$

In above eqs.(6a)-(6e), the symbols denoted by $H_{op}, H_{oe}, H_{ep}, H_D$, and H_A are harmonic Hamiltonian, electron Hamiltonian, electron phonon interaction Hamiltonian, defect Hamiltonian, and anharmonic Hamiltonian respectively. $b_q^{*}\left(b_q\right)$, $a_k^{*}\left(a_k\right)$, ε_q , ε_k , g, $D\left(\vec{k}_1, \vec{k}_2\right), C\left(\vec{k}_1, \vec{k}_2\right), V^{(s)}\left(\vec{k}_1, \vec{k}_2, \ldots, \vec{k}_s\right)$ are creation (annihilation) operators of electron, creation (annihilation) operators of phonon, electron band energy, phonon frequency (in energy units), electron phonon coupling constant, force constant change parameter, mass change parameter, and Fourier transforms of atomic force constants respectively [5,15-25].

This study is done by taking electron Green`s function. An equation of motion technique of quantum dynamics and Dyson equation approach is applied to obtain Fourier transformed electron Green`s function $G_{qq`}\left(\varepsilon\right)$ as [26, 22, 23, 11-13, 5]

$$G_{qq`}\left(\varepsilon\right) = \left( \delta_{qq`} \big/ \pi \right) \left[\left(\varepsilon - \tilde{\varepsilon}_q\right) + i\Gamma\left(kqQ, \varepsilon\right) \right]^{-1} \qquad (7)$$

In eq.(7), $\tilde{\varepsilon}_q$, and $\Gamma\left(kqQ, \varepsilon\right)$ are perturbed mode energy, and electron phonon linewidth respectively. The perturbed mode energy $\tilde{\varepsilon}_q$ is found as

$$\tilde{\varepsilon}_q = \varepsilon_q + \Delta\left(kqQ, \varepsilon\right) \qquad (8)$$

Where in eq.(8), $\Delta\left(kqQ, \varepsilon\right)$ denotes electron phonon shift .

The response function $P\left(kqQ, \varepsilon\right)$ is written as

$$P\left(kqQ, \varepsilon, \omega\right) = \Delta\left(kqQ, \varepsilon\right) - i\Gamma\left(kqQ, \varepsilon\right) \ , \ \omega \to 0^{+} \qquad (9)$$

An electron phonon shift $\Delta\left(kqQ, \varepsilon\right)$ is obtained as

$$\Delta\left(kqQ, \varepsilon\right) = \Delta_{oep}\left(kqQ, \varepsilon\right) + \Delta_{oe}\left(kqQ, \varepsilon\right) + \Delta_{D}\left(kqQ, \varepsilon\right) + \Delta_{De}\left(kqQ, \varepsilon\right) + \Delta_{3A}\left(kqQ, \varepsilon\right) + \Delta_{3Ae}\left(kqQ, \varepsilon\right) + \Delta_{4A}\left(kqQ, \varepsilon\right) + \Delta_{4Ae}\left(kqQ, \varepsilon\right) \qquad (10)$$

Each terms of eq.(10) are found as

$$\Delta_{oep}\left(kqQ, \varepsilon\right) = 4g^2 N_Q \left[\varepsilon_{-kq}{}^2 X_{kq}{}^2 (-)\left(\varepsilon - \varepsilon_k\right)^{-1} - \varepsilon_{+kq}{}^2 X_{kq}{}^2 (+)\left(\varepsilon + \varepsilon_k\right)^{-1} - 2\varepsilon_k X_{kq}{}^2 \{ \varepsilon_q \right.$$

$$\left(\varepsilon_k{}^2 - 3\varepsilon_q{}^2\right)X_{kq}{}^2\left(\varepsilon-\varepsilon_q\right)^{-1} - \varepsilon_k{}^2\left(\varepsilon-\varepsilon_q\right)^{-2}\Big\}\Big]_P \quad (11a)$$

$$\Delta_{oe}(kqQ,\varepsilon) = \alpha g^2\left[X_{qQ}{}^2(+)\left\{\left(\varepsilon+\varepsilon_Q\right)^{-1} - \left(\varepsilon-\varepsilon_q\right)^{-1}\right\} + \left(\varepsilon_q+\varepsilon_Q\right)^{-1}\left(\varepsilon-\varepsilon_q\right)^{-2}\right]_P \quad (11b)$$

$$\Delta_D(kqQ,\varepsilon) = 16g^2 N_Q \sum_{k_1}\left|D\left(\vec{k}_1,-\vec{k}\right)\right|^2\left[X_{k_1q}{}^2(+)\left(\varepsilon-\varepsilon_{k_1}\right)^{-1} - X_{k_1q}{}^2(-)\left(\varepsilon+\varepsilon_{k_1}\right)^{-1}\right]_P \quad (11c)$$

$$\Delta_{De}(kqQ,\varepsilon) = -32g^2 N_Q \sum_{k_1}\left|D\left(\vec{k}_1-\vec{k}\right)\right|^2\left[\varepsilon_{k_1}X_{k_1q}{}^2\left\{2\varepsilon_q X_{k_1q}{}^2\left(\varepsilon-\varepsilon_q\right)^{-1} + \left(\varepsilon-\varepsilon_q\right)^{-2}\right\}\right]_P \quad (11d)$$

$$\Delta_{3A}(kqQ,\varepsilon) = 36g^2\pi^{-1}N_Q \sum_{k_1,k_2}\left|V^{(3)}\left(\vec{k}_1,\vec{k}_2,-\vec{k}\right)\right|^2\Big[\eta_1 S_{+1}\Big\{X_{+\alpha q}{}^2(-)\left(\varepsilon-\varepsilon_{+\alpha}\right)^{-1} - X_{+\alpha q}{}^2(+)$$
$$\left(\varepsilon+\varepsilon_{+\alpha}\right)^{-1}\Big\} + \eta_1 S_{-1}\Big\{X_{-\alpha q}{}^2(-)\left(\varepsilon-\varepsilon_{-\alpha}\right)^{-1} - X_{-\alpha q}{}^2(+)\left(\varepsilon+\varepsilon_{-\alpha}\right)^{-1}\Big\}\Big]_P \quad (11e)$$

$$\Delta_{3Ae}(kqQ,\varepsilon) = -72g^2\pi^{-1}N_Q \sum_{k_1,k_2}\left|V^{(3)}\left(\vec{k}_1,\vec{k}_2,-\vec{k}\right)\right|^2\Big[\eta_1\left(S_{+1}\varepsilon_{+\alpha}X_{+\alpha q}{}^2 + S_{-1}\varepsilon_{-\alpha}X_{-\alpha q}{}^2\right)\Big\{2\varepsilon_q$$
$$\left(\varepsilon-\varepsilon_q\right)^{-1} + \left(\varepsilon-\varepsilon_q\right)^{-2}\Big\}\Big]_P \quad (11f)$$

$$\Delta_{4A}(kqQ,\varepsilon) = 32g^2\pi^{-1}N_Q \sum_{k_1,k_2,k_3}\left|V^{(4)}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3,-\vec{k}\right)\right|^2\Big[\eta_2 S_{+2}\Big\{X_{+\beta q}{}^2(-)\left(\varepsilon-\varepsilon_{+\beta}\right)^{-1} - X_{+\beta q}{}^2(+)$$
$$\left(\varepsilon+\varepsilon_{+\beta}\right)^{-1}\Big\} + 3\eta_2 S_{-2}\Big\{X_{-\beta q}{}^2(-)\left(\varepsilon-\varepsilon_{-\beta}\right)^{-1} - X_{-\beta q}{}^2(+)\left(\varepsilon+\varepsilon_{-\beta}\right)^{-1}\Big\}\Big]_P \quad (11g)$$

$$\Delta_{4Ae}(kqQ,\varepsilon) = -64g^2\pi^{-1}N_Q \sum_{k_1,k_2,k_3}\left|V^{(4)}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3,-\vec{k}\right)\right|^2\Big[\eta_2\left(S_{+2}\varepsilon_{+\beta}X_{+\beta q}{}^2 + 3S_{-2}\varepsilon_{-\beta}X_{-\beta q}{}^2\right)$$
$$\Big\{2\varepsilon_q\left(\varepsilon-\varepsilon_q\right)^{-1} + \left(\varepsilon-\varepsilon_q\right)^{-2}\Big\}\Big]_P \quad (11h)$$

"P" in eq.(11a)- eq.(11h) is the principal value.

And,

Electron phonon linewidth $\Gamma(kq,\varepsilon)$ is found as

$$\Gamma(kqQ,\varepsilon) = \Gamma_{oep}(kqQ,\varepsilon) + \Gamma_{oe}(kqQ,\varepsilon) + \Gamma_D(kqQ,\varepsilon) + \Gamma_{De}(kqQ,\varepsilon) + \Gamma_{3A}(kqQ,\varepsilon) +$$
$$\Gamma_{3Ae}(kqQ,\varepsilon) + \Gamma_{4A}(kqQ,\varepsilon) + \Gamma_{4Ae}(kqQ,\varepsilon) \quad (12)$$

Where,

$$\Gamma_{oep}(kqQ,\varepsilon) = 4\pi g^2 N_Q\Big[\varepsilon_{+kq}{}^2 X_{kq}{}^2(+)\delta\left(\varepsilon+\varepsilon_k\right) - \varepsilon_{-kq}{}^2 X_{kq}{}^2(-)\delta\left(\varepsilon-\varepsilon_k\right) + 2\varepsilon_k X_{kq}{}^2$$
$$\Big\{\varepsilon_q X_{kq}{}^2\left(\varepsilon_k{}^2 - 3\varepsilon_q{}^2\right)\delta\left(\varepsilon-\varepsilon_q\right) - \varepsilon_k{}^2\pi^{-1}\,\mathrm{Im.}\left(\varepsilon-\varepsilon_q\right)^{-2}\Big\}\Big] \quad (13a)$$

$$\Gamma_e(kqQ,\varepsilon) = \pi\alpha g^2\left(\varepsilon_q+\varepsilon_Q\right)^{-2}\Big[\left\{\delta\left(\varepsilon-\varepsilon_q\right) - \delta\left(\varepsilon+\varepsilon_Q\right)\right\} + \pi^{-1}\left(\varepsilon_q+\varepsilon_Q\right)\mathrm{Im.}\left(\varepsilon-\varepsilon_q\right)^2\Big] \quad (13b)$$

$$\Gamma_D(kqQ,\varepsilon)=16\pi g^2 N_Q\sum_{k_1}\left|D\left(\vec{k}_1,-\vec{k}\right)\right|^2\left[X_{k_1q}{}^2(-)\delta\left(\varepsilon+\varepsilon_{k_1}\right)-X_{k_1q}{}^2(+)\delta\left(\varepsilon-\varepsilon_{k_1}\right)\right] \quad (13c)$$

$$\Gamma_{De}(kqQ,\varepsilon)=32\pi g^2 N_Q\sum_{k_1}\left|D\left(\vec{k}_1-\vec{k}\right)\right|^2\varepsilon_{k_1}X_{k_1q}{}^2\left[2\varepsilon_q X_{k_1q}{}^2\delta\left(\varepsilon-\varepsilon_q\right)+\pi^{-1}\,\mathrm{Im.}\left(\varepsilon-\varepsilon_q\right)^{-2}\right] \quad (13d)$$

$$\Gamma_{3A}(kqQ,\varepsilon)=36g^2 N_Q\sum_{k_1,k_2}\left|V^{(3)}\left(\vec{k}_1,\vec{k}_2,-\vec{k}\right)\right|^2\eta_1\left[S_{+1}\left\{X_{+\alpha q}{}^2(+)\delta\left(\varepsilon+\varepsilon_{+\alpha}\right)-X_{+\alpha q}{}^2(-)\delta\left(\varepsilon-\varepsilon_{+\alpha}\right)\right\}+\right.$$

$$\left.S_{-1}\left\{X_{-\alpha q}{}^2(+)\delta\left(\varepsilon+\varepsilon_{-\alpha}\right)-X_{-\alpha q}{}^2(-)\delta\left(\varepsilon-\varepsilon_{-\alpha}\right)\right\}\right] \quad (13e)$$

$$\Gamma_{3Ae}(kqQ,\varepsilon)=72g^2 N_Q\sum_{k_1,k_2}\left|V^{(3)}\left(\vec{k}_1,\vec{k}_2,-\vec{k}\right)\right|^2\eta_1\left(S_{+1}\varepsilon_{+\alpha}X_{+\alpha q}{}^2+S_{-1}\varepsilon_{-\alpha}X_{-\alpha q}{}^2\right)\left[2\varepsilon_q\delta\left(\varepsilon-\varepsilon_q\right)+\right.$$

$$\left.\pi^{-1}\,\mathrm{Im.}\left(\varepsilon-\varepsilon_q\right)^{-2}\right] \quad (13f)$$

$$\Gamma_{4A}(kqQ,\varepsilon)=32g^2 N_Q\sum_{k_1,k_2,k_3}\left|V^{(4)}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3,-\vec{k}\right)\right|^2\eta_2\left[S_{+2}\left\{X_{+\beta q}{}^2(+)\delta\left(\varepsilon+\varepsilon_{+\beta}\right)-X_{+\beta q}{}^2(-)\right.\right.$$

$$\left.\left.\delta\left(\varepsilon-\varepsilon_{+\beta}\right)\right\}+3S_{-2}\left\{X_{-\beta q}{}^2(+)\delta\left(\varepsilon+\varepsilon_{-\beta}\right)-X_{-\beta q}{}^2(-)\delta\left(\varepsilon-\varepsilon_{-\beta}\right)\right\}\right] \quad (13g)$$

$$\Gamma_{4Ae}(kqQ,\varepsilon)=64g^2 N_Q\sum_{k_1,k_2,k_3}\left|V^{(4)}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3,-\vec{k}\right)\right|^2\eta_2\left(S_{+2}\varepsilon_{+\beta}X_{+\beta q}{}^2+3S_{-2}\varepsilon_{-\beta}X_{-\beta q}{}^2\right)\left[2\varepsilon_q\right.$$

$$\left.\delta\left(\varepsilon-\varepsilon_q\right)+\pi^{-1}\,\mathrm{Im.}\left(\varepsilon-\varepsilon_q\right)^{-2}\right] \quad (13h)$$

The different symbols of eqs.(13a-13h) are given as

$$\tilde{n}_k=\left\langle A_k B_k\right\rangle\ ;\ \tilde{\tilde{n}}_k=\left\langle B_k{}^* B_k\right\rangle\ ;\ n_k=\left\langle A_k{}^* A_k\right\rangle\ ;\ \varepsilon_{\pm\alpha}=\tilde{\varepsilon}_{k_1}\pm\tilde{\varepsilon}_{k_2}\ ;\ \varepsilon_{\pm\beta}=\tilde{\varepsilon}_{k_1}\pm\tilde{\varepsilon}_{k_2}\pm\tilde{\varepsilon}_{k_3}$$

$$\varepsilon_{\pm kq}=\varepsilon_k{}^2-\varepsilon_q{}^2\pm\varepsilon_q\varepsilon_k \quad (14a)$$

$$\alpha=4\varepsilon_q{}^2\tilde{\tilde{n}}_k+4\varepsilon_q\varepsilon_k\tilde{n}_k+\varepsilon_k{}^2 n_k+72\sum_{k_1,k_2}\left|V^{(3)}\left(\vec{k}_1,\vec{k}_2,-\vec{k}\right)\right|^2 n_{k_1}n_{k_2}+384\sum_{k_1,k_2,k_3}\left|V^{(4)}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3,-\vec{k}\right)\right|^2$$

$$n_{k_1}n_{k_2}n_{k_3}+16\sum_{k_1}\left|D\left(\vec{k}_1,-\vec{k}\right)\right|^2 n_{k_1} \quad (14b)$$

$$S_{\pm1}=n_{k_2}\pm n_{k_1}\ ;\ S_{\pm2}=1\pm n_{k_1}n_{k_2}+n_{k_2}n_{k_3}\pm n_{k_3}n_{k_1}\ ;\ N_Q=\left\langle b_Q{}^* b_Q\right\rangle\ ;\ n_k=\left(\tilde{\varepsilon}_k/\varepsilon_k\right)Coth\left(\beta\hbar\varepsilon_k/2\right) \quad (14c)$$

$$\eta_{i-1}=\varepsilon_{k_1}\varepsilon_{k_2}\ldots\varepsilon_{k_i}\Big/\tilde{\varepsilon}_{k_1}\tilde{\varepsilon}_{k_2}\ldots\tilde{\varepsilon}_{k_i}\ :\ X_{ij}{}^2(\pm)=\left(\varepsilon_i\pm\varepsilon_j\right)^{-2}\ ;\ X_{ij}{}^2=\left(\varepsilon_i{}^2-\varepsilon_j{}^2\right)^{-1}$$

$$i=k,k_1,\pm\alpha,\pm\beta,q\quad ;\quad j=q,Q \quad (14d)$$

4. Analysis of First order Raman Spectra of Germanium

The density of states in Lehman representation, on taking Fourier transformed electron Green`s function into consideration, is given as[27]

$$N_{qq'}(\varepsilon) = -\sum_{q} \mathrm{Im}.G_{qq'}(\varepsilon + i\omega) \tag{15}$$

The density of states eq.(15) which represents spectral density function, with the help of eq.(7), can be written as

$$N_{qq'}(\varepsilon) = \Gamma_{ep}(kqQ,\varepsilon) \Big/ \pi\left[\left(\varepsilon - \tilde{\varepsilon}_q\right)^2 + \Gamma_{ep}{}^2(kqQ,\varepsilon)\right] \tag{16}$$

Let us take the first order Raman scattering in which zone centre $\vec{k} = 0$ optical phonons are responsible for participation. An eq.(16), with the help of eq.(8) and in the condition $\varepsilon_q \cong \varepsilon_0$, can be taken as

$$N_{qq'} = \Gamma_{ep}(0,\varepsilon) \Big/ \pi\left[\left\{\varepsilon - \left(\varepsilon_0 + \Delta(0,\varepsilon)\right)\right\}^2 + \Gamma_{ep}{}^2(0,\varepsilon)\right] \tag{17}$$

Eq.(17) interpret that equation of motion technique gives the same spectral density function as given by eq.(3) [14].

In the high temperature limit, only cubic and quartic anharmonic terms of electron phonon shift are taken into consideration. They can be solved for $\vec{k} = 0$ as [28]

$$\Delta_{3A}(0,q,\varepsilon) \cong 36g^2\pi^{-1}N_Q \sum_{k_1,k_2} \left|V^{(3)}\left(\vec{k}_1,\vec{k}_2,0\right)\right|^2 \left[\eta_1 S_{+1} X_{+\alpha q}{}^2(-)\left(\varepsilon - \varepsilon_{+\alpha}\right)^{-1}\right]_P$$

$$\cong O_1 T \tag{18a}$$

$$\Delta_{3Ae}(0,q,\varepsilon) \cong -144g^2\pi^{-1}N_Q \sum_{k_1,k_2} \left|V^{(3)}\left(\vec{k}_1,\vec{k}_2,0\right)\right|^2 \left[\eta_1 S_{+1}\varepsilon_{+\alpha} X_{+\alpha q}{}^2 \varepsilon_q \left(\varepsilon - \varepsilon_q\right)^{-1}\right]_P$$

$$\cong O_2 T \tag{18b}$$

The total contribution of cubic anharmonic term to electron phonon shift is

$$\Delta^{3A} \cong (O_1 + O_2)T \cong RT \tag{19}$$

And,

$$\Delta_{4A}(0,q,\varepsilon) \cong 32g^2\pi^{-1}N_Q \sum_{k_1,k_2,k_3} \left|V^{(4)}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3,0\right)\right|^2 \left[\eta_2 S_{+2} X_{+\beta q}{}^2(-)\left(\varepsilon - \varepsilon_{+\beta}\right)^{-1}\right]_P$$

$$\cong S_1 T^2 \tag{20a}$$

$$\Delta_{4Ae}(0,q,\varepsilon) \cong -128g^2\pi^{-1}N_Q \sum_{k_1,k_2,k_3} \left|V^{(4)}\left(\vec{k}_1,\vec{k}_2,\vec{k}_3,0\right)\right|^2 \left[\eta_2 S_{+2}\varepsilon_{+\beta} X_{+\beta q}{}^2 \varepsilon_q \left(\varepsilon - \varepsilon_q\right)^{-1}\right]_P$$

$$\cong S_2 T^2 \tag{20b}$$

The total contribution of quartic anharmonic term to electron phonon shift is

$$\Delta^{4A} \cong (S_1 + S_2)T \cong UT^2 \qquad (21)$$

The combined contribution due to cubic anharmonic term eq.(19) and quartic anharmonic term eq.(21) for electron phonon shift is obtained as

$$\Delta \cong RT + UT^2 \qquad (22)$$

In high temperature, only cubic and quartic anharmonic terms of electron phonon linewidth are taken into account. They can be solved for $\vec{k} = 0$ [28]

$$\Gamma_{3A}(0,q,\varepsilon) \cong 36g^2 N_Q \sum_{k_1,k_2} \left|V^{(3)}(\vec{k}_1,\vec{k}_2,0)\right|^2 \eta_1 S_{+1} X_{+\alpha q}{}^2(+)\delta(\varepsilon+\varepsilon_{+\alpha})$$

$$\cong V_1 T \qquad (23a)$$

$$\Gamma_{3Ae}(0,q,\varepsilon) \cong 72g^2 N_Q \sum_{k_1,k_2} \left|V^{(3)}(\vec{k}_1,\vec{k}_2,-\vec{k})\right|^2 \eta_1 S_{+1}\varepsilon_{+\alpha} X_{+\alpha q}{}^2 \left[2\varepsilon_q \delta(\varepsilon-\varepsilon_q) + \pi^{-1}\,\mathrm{Im.}(\varepsilon-\varepsilon_q)^{-2}\right]$$

$$\cong V_2 T \qquad (23b)$$

The total contribution of cubic anharmonic term to electron phonon linewidth is

$$\Gamma^{3A} \cong (V_1 + V_2)T \cong WT \qquad (24)$$

$$\Gamma_{4A}(0,q,\varepsilon) \cong 32g^2 N_Q \sum_{k_1,k_2,k_3} \left|V^{(4)}(\vec{k}_1,\vec{k}_2,\vec{k}_3,0)\right|^2 \eta_2 S_{+2} X_{+\beta q}{}^2(+)\delta(\varepsilon+\varepsilon_{+\beta})$$

$$\cong Y_1 T^2 \qquad (25a)$$

$$\Gamma_{4Ae}(0,q,\varepsilon) \cong 128g^2 N_Q \sum_{k_1,k_2,k_3} \left|V^{(4)}(\vec{k}_1,\vec{k}_2,\vec{k}_3,0)\right|^2 \eta_2 \varepsilon_q S_{+2}\varepsilon_{+\beta} X_{+\beta q}{}^2 \delta(\varepsilon-\varepsilon_q)$$

$$\cong Y_2 T^2 \qquad (25b)$$

The total contribution of quartic anharmonic term to electron phonon linewidth is

$$\Gamma^{4A} \cong (Y_1 + Y_2)T \cong ZT^2 \qquad (26)$$

The combined contribution of cubic anharmonic term eq.(24) and quartic anharmonic term eq.(26) for electron phonon linewidth is obtained as

$$\Gamma \cong WT + ZT^2 \qquad (27)$$

An electron phonon shift eq.(22) and electron phonon linewidth eq.(27) have got same temperature variation as that of Balkanski et al. [29] at high temperature.

The Raman intensity eq.(4) can be modify by the eq.(22) and eq.(27) as

$$I = P(WT + ZT^2) \Big/ \pi\left[\left\{\varepsilon - (\varepsilon_0 + RT + UT^2)\right\}^2 + (WT + ZT^2)^2\right] \qquad (28)$$

The values of P, ε_0, W, Z, R, and U of eq.(28) are given in the following table-1 with corresponding references from which they are evaluated.

Table-1(CONSTANTS USED IN THE ANALYSIS)

P[30]	ε_0 (1/Cm.) [31]	W(1/Cm.K) [32]	Z(1/Cm.K^2) [32]	R(1/Cm.K)[31]	U(1/Cm.K^2)[31]
35.552129	305.974876	0.01493903	0.0000016	-0.01175793	-0.00000954

The evaluated values of P, ε_0, W, Z, R, and U of table-1 are substituted in eq.(28) to give the Raman spectra with respect to frequency ε in frequency range (297.4 cm^{-1}-309 cm^{-1}) at temperature $10^0 K$ for germanium crystal (figure-1).

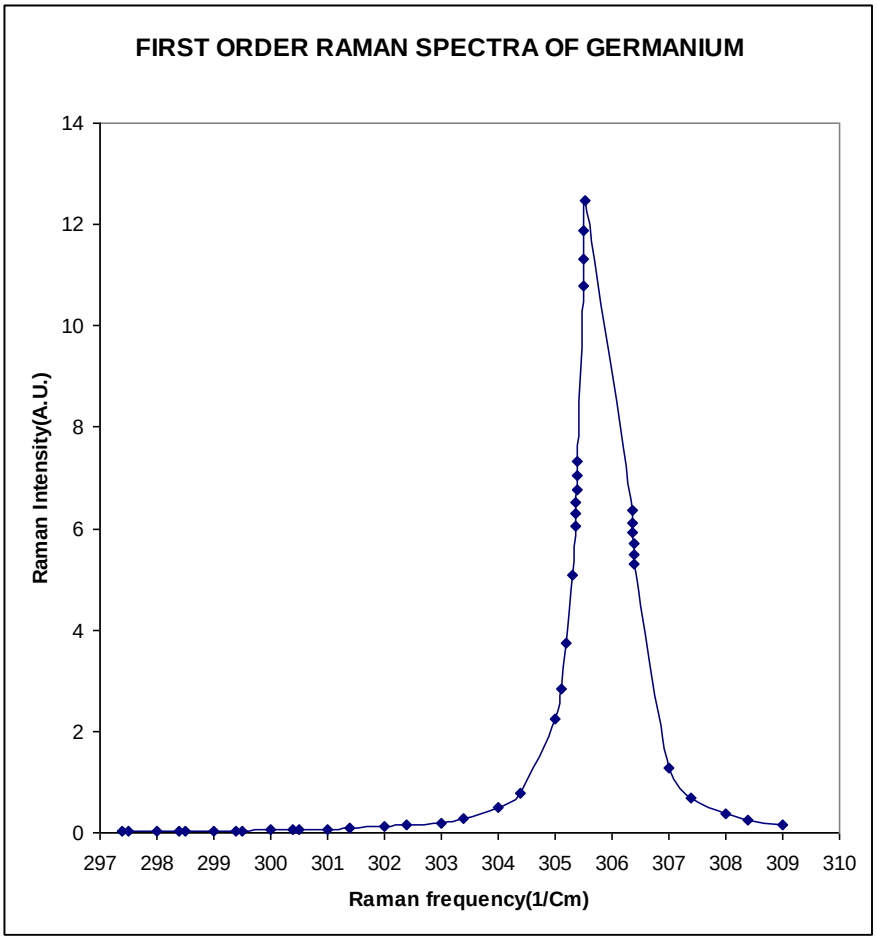

Figure-1(VARIATION OF RAMAN INTENSITY (I) WITH FREQUENCY (ε))

5. Conclusion

The response function obtained from equation of motion technique gives various information through electron phonon linewidth and electron phonon shift. These width and shift are dependent on harmonic field, electron field, localized field, and anharmonic field. In this field

exciton state peak , polaron state peak, one phonon bound state peak, localized state peak, renormalized two phonon and renormalized three phonon combination and difference bands peaks are depended as g^2 on electron phonon coupling constant. It has been found that when two and three phonon renormalized combination and difference mode are identical with exciton state, then strength of Raman intensity is increased. This is due to the creation of exciton state. At high temperature limit, electron phonon linewidth and electron phonon shifts are vary as T, T^2 with temperature through cubic and quartic anharmonicity respectively. This dependence of linewidth and shift on temperature matches with the work of Balkanski et al.[29] at high temperature. This work interpret that combination and difference bands, cubic-order and quartic-order of atomic force constants are the responsible factor of these types of temperature dependence. The shift increases when frequency is identical with one phonon bound state, localized state, combination bands, difference bands and exciton state. The analysis has been done on natural germanium crystal with the help of P, ε_0, W, Z, R, and U constants of table-1 in the frequency range 297.4 cm^{-1}- 309 cm^{-1} at 10^0K (Figure1). The graph of Raman intensity for natural germanium shows that it decreases on both sides of the calculated peak. This variation is just similar to work carried out by earlier authors[33].

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