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MICROPOLAR FLUID OVER A STRETCHING SURFACE IN A NON-DARCIAN POROUS MEDIUM WHEN VISCOSITY AND THERMAL CONDUCTIVIYT VARY WITH TEMPERATURE IN PRESENCE OF MAGNETIC FIELD

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ABSTRACT

The micropolar fluid flow over moving stretching surface in a non-Darcian porous medium with a uniform magnetic field is examined when viscosity and thermal conductivity are vary with temperature. Both the fluid viscosity and thermal conductivity are considered as an inverse linear function of temperature. Mathematical formulation of the problem under consideration is presented and a similarity transformation is applied to reduce the system of partial differential equations and their boundary conditions, describing the problem, into a boundary value problem of ordinary differential equations. The system of equations is solved numerically by shooting technique. The results are presented graphically for velocity, temperature and micropolar distributions for various values of non-dimensional parameters.

Key Words: Viscosity, Thermal conductivity, Micropolar fluid, Porous media.

1. INTRODUCTION:

The theory of micro polar fluids was originally formulated by Eringen ([4], [5]). In essence, the theory introduces new material parameters, an additional independent vector field, the micro rotation and new constitutive equations, which must be solved simultaneously with the usual equations for Newtonian flow. The desire to model the non-Newtonian flow of fluid containing rotating micro-constituents provided initial motivation for the development of the theory, but subsequent studies have successfully applied the model to a wide range of applications including blood flow, lubricants, porous media, turbulent shear flows and flowing capillaries and micro channels by Lukaszewiesz [9].

The theory of thermo-micropolar fluids has been developed by Eringen taking into account the effect of micro-elements of fluids on both the kinematics and conduction of heat. Later Ariman et al. [3] describe some of the various applications which have been explored. Boundary layer on continuous surface is an important type of flow occurring in a number of technical problems. Karwe and Jaluria[7] carried out a numerical study of the transport arising due to the movement of a continuous heated body. The boundary layer flow of a micropolar fluid past a semi-infinite plate has been studied by Peddieson and McNitt [10] where as a similarity solution for boundary layer flow near stagnation point was presented by Ebert [6]. The boundary layer flow of micropolar fluids past a semi infinite plate was studied by Ahmadi[1] taking into account the gyration vector normal to the xy- plane and the micro-inertia effects. By drawing the continuous strips through a quiescent electrically conducting fluid subject to a magnetic field, the rate of cooling can be controlled and final product of desired characteristics can be achieved. The theory of lubrication for micropolar fluid was studied by Allen and Kline[2]. Kelson and Farrell [8] studied micropolar flow over a stretching sheet with strong suction and injection.

Flow and heat transfer through porous media have several practical engineering applications such as transpiration cooling, packed bed chemical reactors, geothermal systems, that the radiation effect is important under many non isothermal situations. If the entire system involving the polymer extrusion process is placed in a thermally controlled environment, then radiation could become important. The knowledge of radiation heat transfer in the system can perhaps lead to a desired product with sought characteristic. The problem of micropolar fluid flow over a stretching surface through a fluid saturated porous medium in presence of magnetic field is therefore an important one.

2. GOVERNING EQUATIONS:

The equation of motion for incompressible viscous micropolar fluid is given by

$$\rho\left\{\frac{\partial\vec{V}}{\partial t}+\left(\vec{V}.\nabla\right)\vec{V}\right\}=-\nabla p+\nabla\left(\mu\nabla.\vec{V}\right)+\kappa\nabla^2\vec{V}+\kappa\left(\nabla\times\vec{N}\right)+\vec{F}\,, \tag{2.1}$$

where ρ is the mass density of the fluid, p is the pressure, μ is the viscosity, $\vec{N}$ is the angular velocity, κ is the material constant and t denotes time. $\vec{F}$ is the body force per unit volume due to flow through porous media given by

$$\vec{F} = \frac{\nu}{\lambda^*}\vec{V},$$

where ν is the kinematic viscosity of the fluid and λ^* is the coefficient of permeability of the porous media.

The equation of angular momentum for incompressible viscous micropolar fluid is given by

$$\rho j\left\{\frac{\partial \vec{N}}{\partial t} + \left(\vec{V}.\nabla\right)\vec{N}\right\} = -2\kappa\vec{N} + \kappa\left(\nabla\times\vec{V}\right) - \gamma\left\{\nabla\times\left(\nabla\times\vec{N}\right)\right\}, \tag{2.2}$$

where j is the micro-inertia per unit mass, γ is the material constants. The equation of heat transfer is given by

$$\rho C_p\left\{\frac{\partial T}{\partial t} + \left(\vec{V}.\nabla\right)T\right\} = \nabla.\left(\lambda\nabla T\right) + \left(\mu+\kappa\right)\phi, \tag{2.3}$$

where C_p is specific heat at constant pressure, T is the temperature of the fluid, λ is the coefficient of thermal conductivity of the fluid, ϕ is the viscous dissipation function and is given by

$$\phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial z}\right)^2\right] + \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right)^2 + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)^2, \tag{2.4}$$

3. MATHEMATICAL FORMULATION OF THE PROBLEM:

Consider a steady, two-dimensional laminar flow of an incompressible micropolar fluid over a stretching surface in a non-Darcian porous medium which issues from a thin slit. The x-axis is taken along the stretching surface and y-axis is perpendicular to it. A uniform magnetic field B_0 is imposed along y-axis. Under the usual boundary layer approximations, the flow and heat transfer of a micropolar fluid in porous medium with the non-Darcian effects included are governed by the following equations.

The equation of continuity is

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{3.1}$$

The equation of momentum is

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+k_1\frac{\partial N}{\partial y}+k\left(\frac{\partial^2 u}{\partial y^2}\right)-\rho\phi u^2-\sigma B_0{}^2 u \tag{3.2}$$

Equation of angular momentum is

$$\rho j\left(u\frac{\partial N}{\partial x}+v\frac{\partial N}{\partial y}\right)=-k\left(2N+\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial y}\left(\gamma\frac{\partial N}{\partial y}\right) \tag{3.3}$$

Equation of energy is

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=\lambda\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y} \tag{3.4}$$

where μ is the coefficient of dynamic viscosity, γ is the apparent kinematic viscosity, N is the microrotation component, S is a constant characteristic of the fluid, $k_1=\frac{S}{\rho}(>0)$ is the coupling constant, ρ is the fluid density, C_p is the specific heat at constant pressure, u and v are the components of velocity along x and y directions respectively, ϕ is the porosity, k is the permeability of the porous medium, T is the temperature of the fluid in the boundary layer, T_∞ is the temperature of the fluid far away from the plate, T_w is the temperature of the plat, λ is the thermal conductivity, σ is the electrical conductivity, B_0 is an external magnetic field and q_r is the radiative heat flux.

The appropriate physical boundary conditions of equations are

$$y=0: \quad u=ax, \quad v=0, \quad T=T_w, \quad N=0 \tag{3.5}$$

$$y \to \infty : u \to 0, T \to T_\infty, \; N \to 0 \tag{3.6}$$

The governing equations subject to the boundary conditions can be expressed in a simpler form by introducing the following transformations:

$$\eta = \sqrt{\frac{a}{\nu}}y, \; \psi = \sqrt{a\nu}xf, \; N = \sqrt{a^3\frac{1}{\nu}}xg, \; \theta = \frac{T - T_\infty}{T_w - T_\infty} \tag{3.7}$$

using the Rosselant approximation , we have

$$q_r = \left(-\frac{4\sigma_0}{3k_0}\right)\frac{\partial T^4}{\partial y}$$

where σ_0 is the Stefan-Boltzmann constant and k_0 is the mean absorption coefficient. The fluid viscosity is assumed to be inverse linear function of temperature (Lai and Kulacki[11]) as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left[1 + \alpha\left(T - T_\infty\right)\right], \frac{1}{\mu} = a\left(T - T_r\right), \; a = \frac{\alpha}{\mu_\infty} \text{ and } T_r = T_\infty - \frac{1}{\alpha} \tag{3.8}$$

where a and T_r are constants and their values depends on the reference state and the thermal property of the fluid. In general $a>0$ for liquids and $a<0$ for gases. T_r is transformed reference temperature related to viscosity parameter. α is constant based on thermal property and μ_∞ is the viscosity at $T=T_\infty$ similarly, consider the variation of thermal conductivity as,

$$\frac{1}{\lambda} = \frac{1}{\lambda_\infty}\left[1 + \xi\left(T - T_\infty\right)\right], \frac{1}{\lambda} = b\left(T - T_k\right), \; b = \frac{\xi}{\lambda_\infty} \text{ and } \; T_k = T_\infty - \frac{1}{\xi} \tag{3.9}$$

where b and T_k are constants and their values depends on the reference state and thermal property of the fluid ξ is constant based on thermal property and λ_∞ is the thermal conductivity at $T=T_\infty$.

Using equation (3.7), it can be easily verified that the continuity equation is satisfied automatically and using equation (3.7) - (3.9) in the equation (3.2) - (3.4) become,

$$f''' = D_a^{-1} f' + \left(Mf' + f'^2 - Kg'\right)\frac{\theta_r - \theta}{\theta_r} - \frac{1}{\theta_r - \theta} f''\theta' \qquad (3.10)$$

$$g'' = \frac{2K}{2+K}\left(2g + f''\right) + \frac{2}{2+K}\left(f'g - fg'\right) \qquad (3.11)$$

$$\theta'' = P_r f \frac{\theta_k - \theta}{\theta_k} + \frac{\theta'}{\theta_k - \theta} + 4\Pr\left[\frac{\theta_k - \theta}{\theta_k}(\theta+1)^2 \theta'^2 + \frac{1}{3}\frac{\theta_k - \theta}{\theta_k}(\theta+1)^3 \theta'\right] \qquad (3.12)$$

The transform boundary conditions are

$$\eta = 0, \quad f' = 1, \quad f = 0, \quad g = 0, \quad \theta = 1 \qquad (3.13)$$

$$\eta = \infty, \quad f' = 0, \quad g = 0, \quad \theta = 0 \qquad (3.14)$$

where

$K = \frac{k_1}{\mu}$, the coupling constant parameter

$D_a^{-1} = \frac{\varphi \nu}{ka}$ the inverse Darcy number

$M = \frac{\sigma_0 B_0^2}{\rho a}$, the magnetic parameter

$G = \frac{\gamma a}{\kappa \nu}$, the microrotation parameter

$P_r = \frac{\nu \rho C_p}{k}$, the Prandtl number

4. RESULTS AND DISCUSSION:

Initially solution was taken for constant values of Pr=0.70 ,M=0.80, G=0.50, Im1=0.50, D_a^{-1}=0.5, r=0.30, θ_r=-10.00 with the viscosity parameter θr ranging from -15 to -1 at the certain values of θ_k=-10. Similarly the solutions have been found with varying the thermal conductivity parameter θ_k ranging from -15 to -1 at the certain values of θr=-10 keeping other values remaining same. We have considered in some detail the influence of the physical parameters D_a^{-1}, *Pr*, *M* , θ_k on the velocity, micro rotation and temperature distributions which shown in figures (1-6). Figures (5) and (6) show the velocity and microrotation for various values of the magnetic parameter *M* respectively. Application of a transverse magnetic field normal to the flow direction gives rise to a resistive drag-like force acting in a direction opposite to that of flow. This has a tendency to reduce both the fluid velocity and angular velocity .This indicates that the velocity and microrotation distribution decreases with the increasing values of *M*. Figures (3) and (4) display the influence of the inverse Darcy number D_a^{-1} on the velocity and the micro rotation profiles respectively. It is obvious that the presence of porous medium causes higher restriction to the fluid. From (3) ,It is seen that the velocity distribution increases with the increasing values of D_a^{-1}. From (4) ,it is seen that the micro rotation distribution decreases then increases with the increasing values of D_a^{-1}. Figures (1) and (2) depict the influence of the thermal conductivity parameter θ_k, Prandtl number *Pr* on the temperature distributions respectively. From figure (1), it is observed that the temperature distributions increases with the increasing values of thermal conductivity parameter θ_k. From (2) it is seen that the temperature distributions decreases as *Pr* increases.

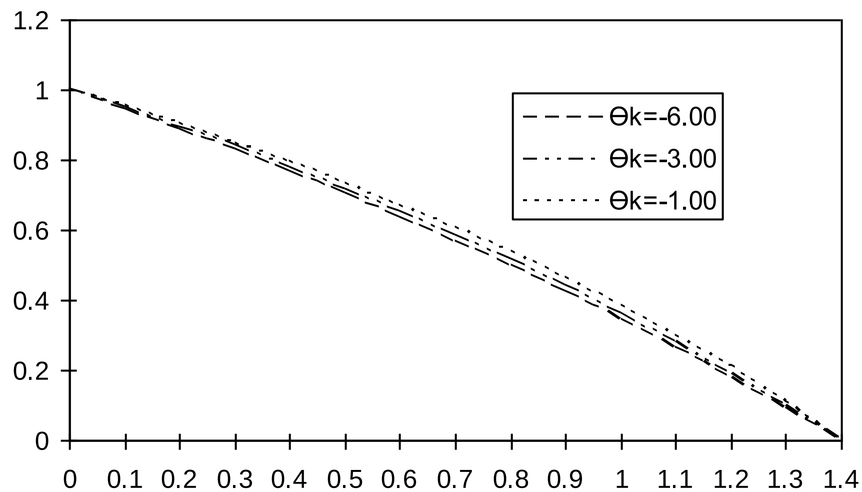

Fig. 1. Temperature distribution profiles along against η for various values of parameter θ_k taking Pr=0.70, M=0.80, G=0.50, lm1=0.50, D_a^{-1}=0.5, r=0.30, θ_r=-10.00

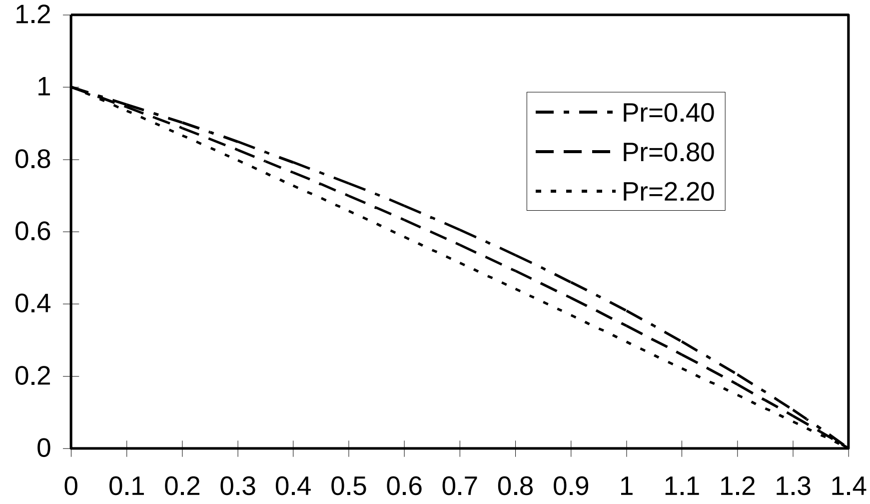

Fig. 2. Temperature distribution profiles along against η for various values of parameter Pr taking M=0.80, G=0.50, Im1=0.50, r=0.30, θ_r =-10.00, θ_k =-10.00

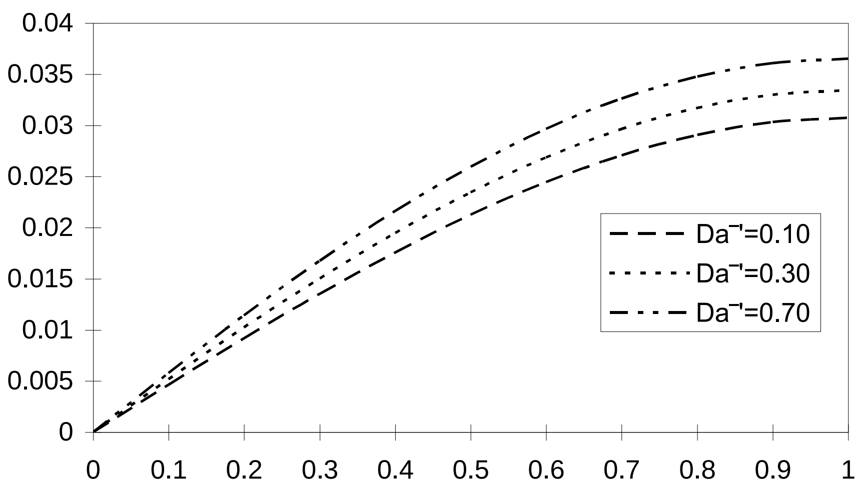

Fig. 3.Velocity distribution profiles along against η for various values of parameter D_a^{-1} taking Pr=0.70,M=0.80, G=0.50, r=0.30, θ_r =-10.00, θ_k =-10.00

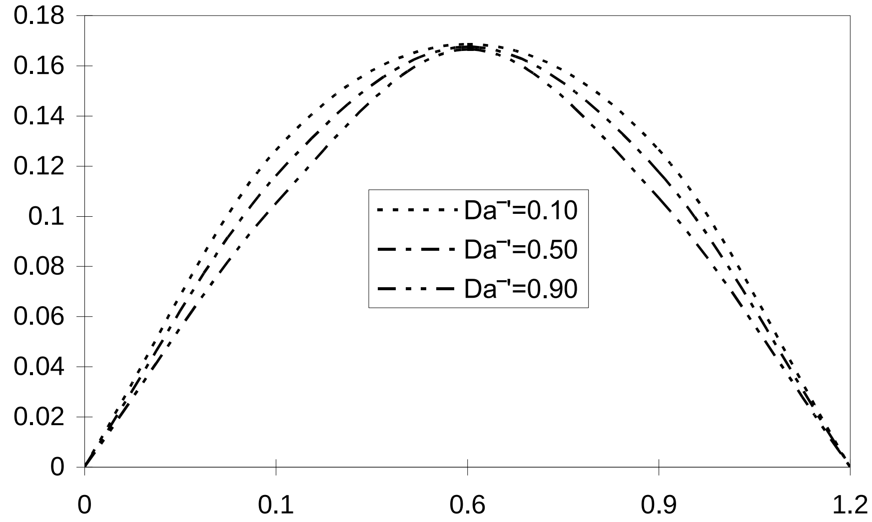

Fig. 4. Microrotation distribution profiles along against η for various values of parameter D_a^{-1} taking Pr=0.70,M=0.80, G=0.50, r=0.30, θ_r =-10.00, θ_k =-10.00

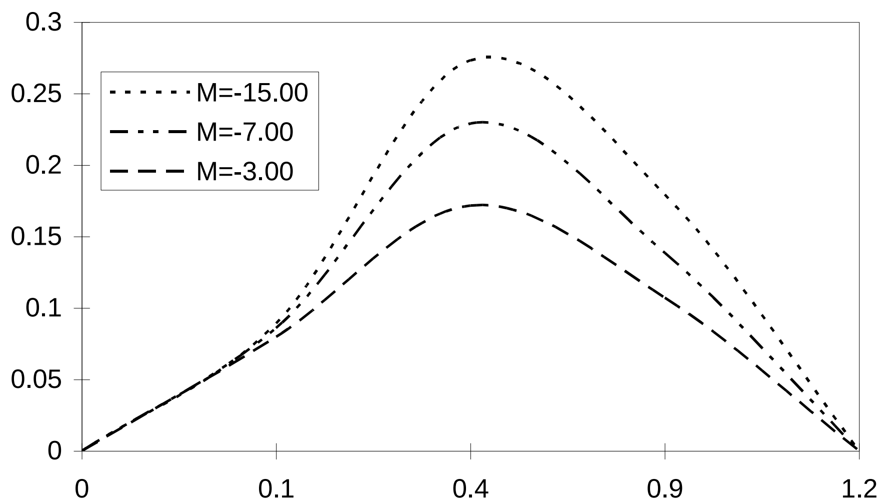

Fig. 5. Velocity distribution profiles along against η for various values of parameter M taking Pr=0.70, G=0.50, Im1=0.50, r=0.30, θ_r =-10.00, θ_k =-10.00

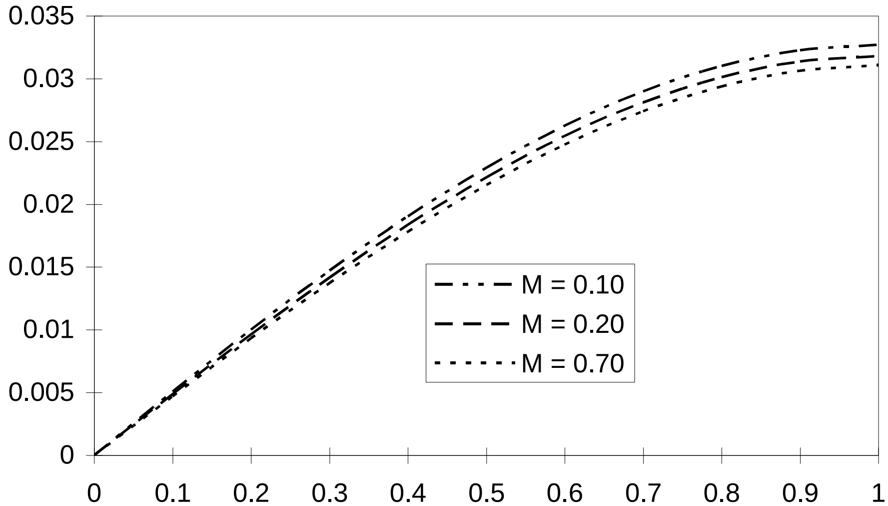

Fig. 6. Microrotation distribution profiles along against η for various values of parameter D_a^{-1} taking Pr=0.70, M=0.80, G=0.50, r=0.30, θ_r =-10.00, θ_k =-10.00

5. CONCLUSION:

The micropolar fluid flow over moving stretching surface in a non-Darcian porous medium with a uniform magnetic field is examined when viscosity and thermal conductivity are vary with temperature is examined. The resulting partial differential equations, which describe the problem, are transformed into ordinary differential equations by using similarity transformations. Numerical evaluations are performed and graphical results are obtained. The results presented demonstrate clearly that the viscosity and thermal conductivity parameters have a substantial effect on velocity distribution, micropolar distribution and temperature distribution. The effect of Darcy number D_a^{-1}, magnetic parameter M, Prandtl number P_r are quite significant.

6. **REFERENCES:**

[1] Ahmadi, G., Self-similar solution of imcompressible micropolar boundary layer flow over a semi-infinite plate, international Journal of Engineering Science, 14,(1976),no7,639-646.

[2] Allen, S.J. and Kline ,K.A., Lubrication theory for micropolar fluids, Journal of Applied Mechanics , Transactions of the ASME ,38,(1971),no.3,646-650.

[3] Ariman T., Turk M.A. and Sylvester N.D Microcontinum fluid mechanics ,Int. J.Engng. Sci., 11,(1973) ,905-930.

[4] Eringen, A.C., Theory of micropolar fluids, Journal of Mathematics and Mechanics, 16,(1966),1-18.

[5] Eringen, A. C., J. Math. Anal. Appl., Vol.38, (1972), 480

[6] Ebert F. A similarity solution for the boundary layer flow of a polar fluid. The Chem. Eng. J.,5,(1973),85-92.

[7] Karwe Mukund. V and Jaluria Y. Thermal transport from a heated moving surface. Trans. of ASME J. Heat Transfer ,108, (1986), 728-734.

[8] Kelson N.A. and Farrell T.W Micropolar flow over a porous stretching sheet with strong suction or injection. Int. Comm. Heat Mass Transfer, 28, .(2001), 479-488.

[9] Lukaszewiez,G. Micropolar fluids, theory and applications, Boston, (1999).

[10] Peddieson J. and McNitt R.P, Boundary layer theory for a micropolar fluid , (1970).

[11] Lai, F. C. and Kulacki, F. A., The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous media, Int. J. Heat Mass Transfer ,Vol.33,No.5, (1990), 1028-1031.