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Website: www.aarf.asia Email : editor@aarf.asia , editoraarf@gmail.com

OPTIMAL PRICING AND REPLENISHMENT POLICIES FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH SUBJECTIVE PRICING STRATEGY

Kamlesh A Patel[1,3] and Nilay Mehta[2]

1. Department of Mathematics, Shri U. P. Arts & Smt. M. G. Panchal Science & Shri V. L. Shah Commerce College, India, Gujarat, Pilvai.
2. Department of Mathematics, K. K. Shah Jarodwala Maninagar Science College, India, Gujarat, Ahmedabad.

ABSTRACT

This study extends the work of Soni and Patel (2013) and consider the pricing strategy in subjective manner when the deterioration occurs in the product. The model assumes price sensitive demand when the product has no deterioration and price and time dependent demand when the product has deterioration. This model allows the retailer to determine price based on his experience and offers best policy for selling price and replenishment cycle for the retailer that aims at maximizing the total profit per unit time. The concavity of the objective function is established using numerical example. The effect of different weight on selling price is analyzed and some managerial implications are presented.

Keywords: Inventory, non-instantaneous deterioration, pricing, replenishment

1. Introduction

The traditional inventory models for deteriorating items are developed with common assumption that deterioration of items in inventory starts from the instant of their arrival. However, many items maintain freshness or original condition for a certain period of time. In other words, deterioration occurs after particular period of time. Wu et al. (2006) introduced this notion and investigated inventory model for non-instantaneous deteriorating items with partial backlogging where demand is stock sensitive. This notion has received wide attention by several researchers such as, Ouyang et al. (2006, 2008), Uthayakumar and Geetha (2009), Chang et al. (2010), Chang and Lin (2010), Musa and Sani (2012).

The pricing is an important parameter to consider for being in business. As a result, the models on non-instantaneous deteriorating items developed by Wu et al. (2009), Yang et al. (2009), Maihami and Kamalabadi (2012a, 2012b), Shah et al. (2013) are worth mentioning in this direction. Recently, Soni and Patel (2013) proposed the model considering (1) an imprecise deterioration free time for the product, (2) price sensitive demand prior to deterioration and price and time dependent demand afterward. They argued demand rate decreases as the product losses the usefulness from the original condition over the time. It is observed in practical situation that the retailer generally moderates the selling price as and when the product gets deteriorate to boost the sale and to reduce the deterioration cost.

This study extend the work of Soni and Patel (2013) and consider the pricing strategy in subjective manner when the deterioration occurs in the product. Unlike Soni and Patel (2013), we consider the deterministic deterioration free time. The model considers optimization of total profit per unit time of the retailer with respect to pricing and ordering policy for non-instantaneous deteriorating items with subjective pricing strategy.

The rest of the paper organized as follows. In Section 2, notation and assumptions are defined. Section 3 is on development of mathematical model to maximize the total profit per unit time. Section 4 presents numerical example along with sensitivity analysis. Finally, conclusion is drawn in Section 5.

2. Notation and assumptions

The following notations (similar to Soni and Patel; 2013) and assumptions are used to develop the model. Some additional notations will be introduced later when they are needed.

2.1 Notation

K : The ordering cost per order.

t_d : The length of deterioration free time.

c : The purchasing cost per unit.

p : The selling price per unit.

h : Unit holding cost per unit time.

w	:	weighting coefficient
Q	:	The order quantity.
T	:	Length of replenishment cycle ($t_d \le T$).
θ	:	The deterioration rate of the on-hand inventory over [t_d, T].
$I_1(t)$	:	The inventory level at time t ($0 \le t \le t_d$) in which the product has no deterioration.
$I_2(t)$	:	The inventory level at time t ($t_d \le t \le T$) in which the product has deterioration.
$\Pi(p,T)$	:	The total profit per unit time of inventory system.

2.2 Assumptions

(These assumptions are mainly adopted from Soni and Patel.; 2013)

(1) The inventory system involves single non-instantaneous deteriorating item.

(2) The on-hand inventory deteriorate with constant rate θ, where $0 < \theta < 1$.

(3) Demand rate $D(p, t)$ is a function of the selling price (p) and time. In this paper, we assume different demand rates depending on the length of deterioration free time and defined as

$$D(p,t) = \begin{cases} D(p) & \text{, if } t \le t_d \\ D(p_1)e^{-\delta(t-t_d)}, & \text{if } t \ge t_d \end{cases}$$

where $D(p) = ap^{-b}$, a (> 0) is the scaling factor, b (> 1) is the index of price elasticity, and δ (> θ) is the shape parameter and $p_1 = wp + (1-w)c$ with $0 < w \le 1$.

(4) There is no replacement or repair of deteriorated units during the period under consideration.

(5) Shortages are not allowed to avoid the lost sales.

(6) Replenishment rate is infinite and lead time is zero.

(7) The system operates for an infinite planning horizon.

3. Model Formulation

Following Eq. (1) of Soni and Patel (2013), the differential equations describing the variation of inventory level $I(t)$, at any instant of time $t \in [0, T]$ are as follows:

$$\frac{dI_1(t)}{dt} = -D(p), \quad 0 \le t \le t_d \tag{1}$$

$$\frac{dI_2(t)}{dt} = -\theta I_2(t) - D(p,t), \quad t_d \le t \le T \tag{2}$$

with terminal condition $I_1(0) = Q$ and $I_2(T) = 0$.

The solution to (1)–(2) is,

$$I_1(t) = Q - D(p)t, \quad 0 \le t \le t_d \tag{3}$$

$$I_2(t) = \frac{D(p_1)e^{-\delta(T-t_d)}}{\varepsilon}\left[e^{\delta(T-t)} - e^{\theta(T-t)}\right], \quad t_d \le t \le T \tag{4}$$

where $\varepsilon = \delta - \theta > 0$.

Since $I_1(t) = I_2(t)$ at $t = t_d$, it follows from (4) and (5) that,

$$Q - D(p)t_d = \frac{D(p_1)e^{-\delta(T-t_d)}}{\varepsilon}\left[e^{\delta(T-t_d)} - e^{\theta(T-t_d)}\right]$$

which yields the order quantity Q, as

$$Q = D(p)t_d + \frac{D(p_1)e^{-\delta(T-t_d)}}{\varepsilon}\left[e^{\delta(T-t_d)} - e^{\theta(T-t_d)}\right] \tag{5}$$

Replacing Q in Eq. (3) by the above expression, we have

$$I_1(t) = \frac{D(p_1)}{\varepsilon}\left[1 - e^{-\varepsilon(T-t_d)}\right] + D(p)(t_d - t) \tag{6}$$

The elements comprising total relevant cost and sales revenue per cycle are listed below:

1. The ordering cost is K
2. The holding cost is

$$h\left[\int_0^{t_d} I_1(t)dt + \int_{t_d}^{T} I_2(t)dt\right] = h\left[\frac{D(p)t_d^2}{2} + \frac{D(p_1)\left(1 - e^{-\varepsilon(T-t_d)}\right)t_d}{\varepsilon}\right] + \frac{hD(p_1)e^{-\delta(T-t_d)}}{\varepsilon}\left[\frac{e^{\delta(T-t_d)}}{\delta} - \frac{e^{\theta(T-t_d)}}{\theta}\right]$$

3. The purchase cost is $c\times Q = cD(p)t_d + \frac{cD(p_1)e^{-\delta(T-t_d)}}{\varepsilon}\left[e^{\delta(T-t_d)} - e^{\theta(T-t_d)}\right]$

4. The sales revenue is

$$\int_0^{t_d} pD(p)dt + \int_{t_d}^{T} p_1 D(p_1)e^{-\delta(t-t_d)}dt = pD(p)t_d + p_1 D(p_1)\left[\frac{1-e^{-\delta(T-t_d)}}{\delta}\right]$$

Assembling above cost and revenue components, the total profit per unit time (denoted by Π (*p*, *T*)) is given by

$$\Pi(p,T) = \frac{1}{T}\{\text{Sales revenue} - \text{Ordering cost} - \text{Holding cost} - \text{Purchase cost}\}$$

Thus, the problem is reduced to

$$\begin{aligned}&\text{Maximize } \Pi(p,T)\\ &\text{Subject to } T \ge t_d\end{aligned} \tag{7}$$

Our objective is to determine the optimal selling price and optimal cycle time which maximizes the total profit per unit time. For this, we use following algorithmic procedure to identify the optimal selling price and optimal cycle time.

Algorithm 1:

Step 1: Set $k = 1$ and initialize the value of $p^{(k)} = c + \Delta$, where Δ is a small value.

Step 2: Obtain the value of T_1 by solving $\partial\Pi(p,T)/\partial T = 0$. If $T_1 \ge t_d$ then set $T^{(k)} = T_1$ otherwise set $T^{(k)} = t_d$.

Step 3: Substitute the value of $T^{(k)}$ into $\partial\Pi(p,T)/\partial p = 0$ to obtain the corresponding value of $p^{(k)}$. Set $p^{(k+1)} = p^{(k)}$

Step 5: If $\left|p^{(k+1)} - p^{(k)}\right| \le 10^{-4}$ then set $(p^*,T^*) = (p^{(k+1)},T^{(k)})$ and (p^*,T^*) is the optimal solution. Otherwise set $k = k + 1$ and go to step 2.

Step 6: Calculate $\Pi(p^*,T^*)$ Also find corresponding Q from Eq. (5).

4. Numerical example

Example 1: [Taken from Soni and Patel (2013) and adapted to our model]

K = \$250 per order, c = \$3 per unit, h = \$0.4 per unit per year, t_d = 15/365 year, a = 400,000, b = 2.5, δ = 0.96, θ = 0.1, α = 0.9.

Executing the procedure proposed in Algorithm 1, we obtain p^* = \$5.2671, T^* = 0.1879, $\Pi\left(p^*,T^*\right)$ = \$ 11974.07 and optimal order quantity Q^* = 1236.56 units.

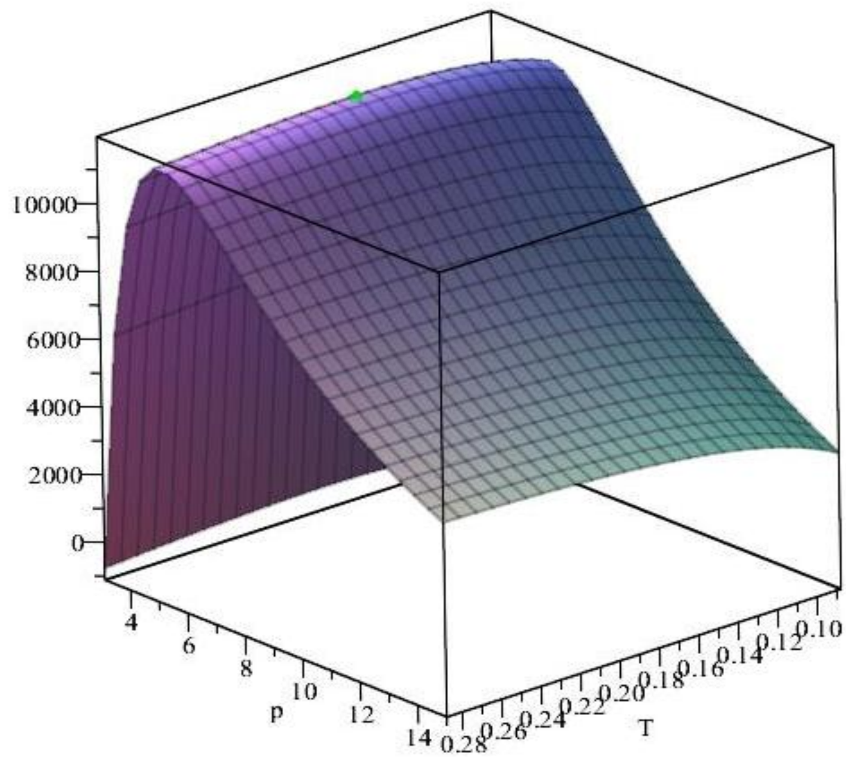

Figure 1: Convexity of $\Pi\left(p,T\right)$ with respect to p and T

Figure 1 depicts the behavior of objective function with respect to selling price (p) and cycle time (T). From Figure 1, it can be seen that the objective function is concave with respect to decision variables.

Example 2: In this example we shall assess the impact of weighting coefficient over the decision variables. Computational results are summarized in Table 1 for various set of values of weighting coefficient.

Table 1: Optimal solution for different values of *w*

w	p^*	T^*	Q^*	$\Pi(p^*, T^*)$
0.1	23.5144	0.3254	54.45	10181.87
0.2	12.4880	0.2804	224.71	10647.51
0.3	9.0650	0.2443	430.54	11061.50
0.4	7.5203	0.2204	613.40	11381.03
0.5	6.6716	0.2058	766.48	11610.33
0.6	6.1362	0.1971	898.80	11768.01
0.7	5.7635	0.1920	1018.27	11872.31
0.8	5.4853	0.1892	1129.87	11937.50
0.9	5.2671	0.1879	1236.56	11974.07
1.0	5.0893	0.1876	1340.15	11989.59

From the Table 1 it can be observed that as the value of weighting coefficient (w) increases, the optimal selling price(p*) and the optimal length of replenishment cycle (T*) whereas the optimal order quantity (Q*) and the optimal profit per unit time increase. These results suggest that the retailer should carefully choose the value of weighting coefficient (w) to obtain maximum return.

5. Conclusion

In this study, we extended the model of Soni and Patel (2013) on non-instantaneous deteriorating item to determine pricing and replenishment policy in subjective manner. This model allows the retailer to determine price based on his experience and offers best policy for selling price and replenishment cycle for the retailer that aims at maximizing the total profit per unit time. The concavity of the objective function is established using numerical example. Furthermore, we illustrated the behavior of our model with respect to key parameters in numerical examples.

The work presented herein could have several possible extensions. For example, this model can be extended to accommodate shortages, variable deterioration rate, stochastic demand, and so forth. One could also incorporate the different preservation investment in the model formulation.

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