



FERROFLUID LUBRICATION FOR ROUGH AND SMOOTH SURFACE SLIDER BEARING WITH AND WITHOUT SQUEEZE VELOCITY

Asha C. S. ¹ and Achala L. Nargund ²

^{1,2}P.G. Department of Mathematics and Research Centre in Applied Mathematics

M.E.S College of Arts, Commerce and Science

15th Cross, Malleswaram, Bangalore-560003.

ABSTRACT

In this paper, the effects of ferrofluid lubricant on hyperbolic slider bearings for smooth and rough surfaces are studied using Jenkin's model. Expressions for pressure and load carrying capacity are obtained.

Keywords: *Ferrofluids, Jenkins model, hyperbolic slider bearing, Stochastic method.*

Mathematics Subject Classification: 76D08

1 Introduction

There are many interesting materials which are not freely available in nature but can be synthesized one of such material is ferrofluid. Ferrofluids are colloidal liquids made of nano scale ferromagnetic or ferrimagnetic particles suspended in a carrier fluid usually an organic solvent (benzene, carbon dioxide, chloroform) or water. The carrier fluid are of two types namely polar and non polar. In polar fluids body torque per unit mass is introduced in addition to the body force and

a couple stress is introduced in addition to the normal stress. Whereas in non polar fluids only body force and Normal stress are introduced. Either the stress tensor is symmetric or the angular momentum is conserved in non polar fluids. In non polar ferrofluid Magnetization \vec{M} is parallel to applied magnetic field \vec{H} . Jenkins model are based on the non polar ferrofluid. Lubrication has a wide range of industrial applications. The use of lubricants in slider bearing is to reduce the friction and increase the precision with load bearing capacity. Many researchers have proposed different lubricants to enhance the load bearing capacity with precise lubrication. Porous slider bearings are used in vacuum cleaners, water pumps, record players, tape recorders and generators. The roller bearings are used in rotary motion, relative motion or combination of both types of motion[12, 13, 6].

Ram and Verma [2] have studied ferrofluid lubrication in porous inclined slider bearing using Jenkins model and showed that the load capacity increases by 13% when all the terms in Jenkin's model are used. Shah and Bhat [3] used ferrofluid lubrication in porous inclined slider bearing with slip velocity and showed that the load capacity, coefficient of friction decreases and the position of the centre of pressure increases as the slip parameter increases. Also showed that when the material constant is increased the friction, coefficient of friction increased and load capacity decreased. Patel et al.[4] analyzed the effect of roughness and slip velocity using rough porous hyperbolic slider bearing for Neuringer-Ronsensweig model and showed that the performance of the bearing is improved for the lower values of slip parameter. Deheri et al. [5] have made an attempt to study the performance of ferrofluid lubrication of a rough porous convex pad slider bearing using Shliomis model and showed that the Shliomis model is more effective than Neuringer-Ronsensweig model. Achala and Asha [6] used Shliomis model for ferrofluid lubrication of hyperbolic slider bearing with smooth surface and observed that the load carrying capacity depends on the curvature of the hyperbolic bearing along with volume concentration and Langevin's parameter.

In the present analysis we are considering Jenkins model with uniform magnetic field using hyperbolic slider bearing for rough and smooth surfaces. In the theory of lubrication of rough bearing surfaces central concept considered is stochastic process. The lubricant film is split up into two parts, one will be nominal film thickness measuring large scale variation in film geometry including any long wavelength disturbances. The other part will be roughness measured from

mean level and is randomly varying. The paper is divided into five sections, section 1 consists of introduction, section 2 consists of geometrical model and governing equations of the problem, section 3 has details of solution, section 4 contains results and discussions, and section 5 has graphs.

2 Geometry and Governing equations

The configuration of the bearing system is shown in fig. 1. We consider smooth and transversely rough bearing surfaces. The lower surface is a slider of length l moving with uniform velocity U in the x direction. h_1 and h_2 are the minimum and maximum film thickness respectively. The thickness of the lubricant film for rough surface is taken as [9, 10, 11]:

$$h(x) = h_{\text{mean}}(x) + h_s, \quad (1)$$

where $h_{\text{mean}}(x)$ is the mean film thickness, h_s is the deviation from the mean film thickness indicating the random roughness of the bearing surfaces. h_s is taken as stochastic in nature and is governed by the probability density function:

$$f(h_s) = \frac{35}{32c} \left(1 - \frac{h_s^2}{c^2} \right), -c \leq h_s \leq c, \text{ elsewhere}$$

where c is the maximum deviation from the mean film thickness.

The film thickness h is defined as

$$h = \frac{h_2}{1 + \left(\frac{x \log a}{l} \right)}, 0 \leq x \leq l, \quad \text{where } a = \frac{h_2}{h_1}.$$

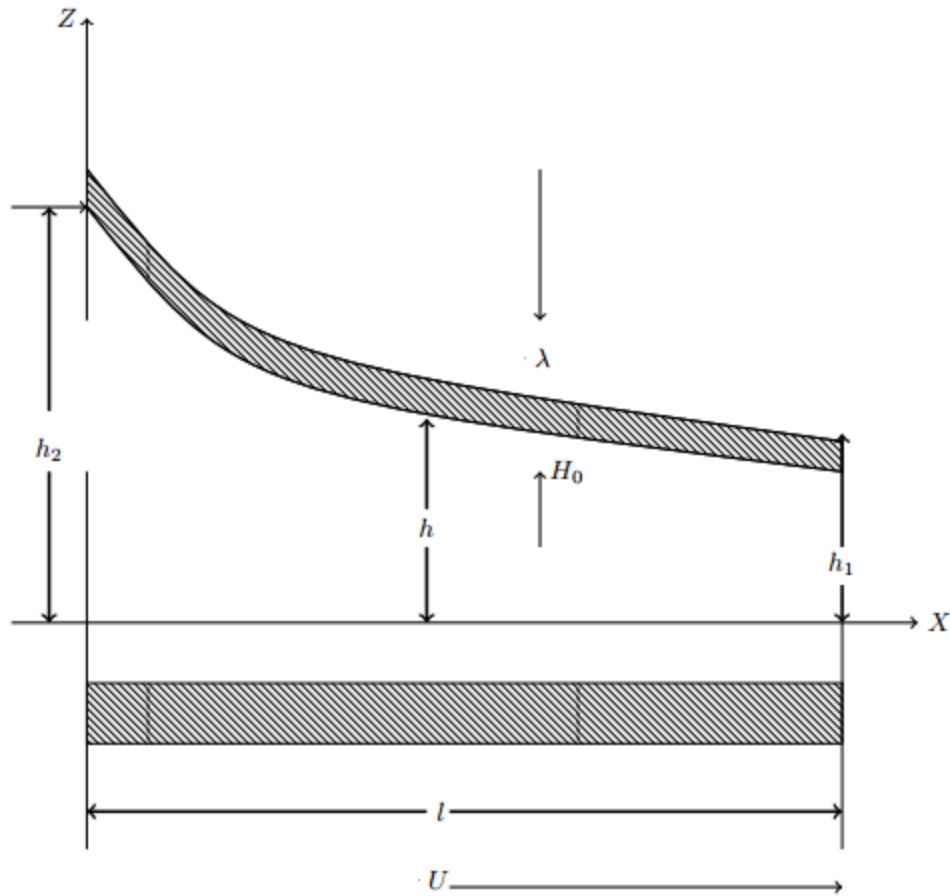


fig. 1. Physical configuration of hyperbolic slider bearing

Consider an incompressible, steady non polar ferrofluid by neglecting inertia. The basic equations governing the Jenkin's model [1] based ferrofluid lubrication are as follows

$$-\nabla \bar{p} + \eta \nabla^2 \bar{q} + \mu_0 (\bar{M} \cdot \nabla) \bar{H} + \frac{\rho \beta^2}{2} \nabla \times \left(\frac{\bar{M}}{M} \times (\nabla \times \bar{q}) \times \bar{M} \right) = 0. \quad (2)$$

$$\nabla \cdot \bar{q} = 0, \bar{M} = \bar{\mu} \bar{H}, \nabla \times \bar{H} = 0, \nabla \cdot (\bar{H} + \bar{M}) = 0, \quad (3)$$

where p is the pressure, \bar{q} is the fluid velocity, \bar{H} is the applied magnetic field, \bar{M} is the Magnetization, ρ is the density, β is the material constant, μ_0 is the permeability of free space, η is the coefficient of viscosity of the fluid.

3 Method of solution

Let us consider a uniform magnetic field

$$\vec{H} = (0,0,H_0), \vec{q} = (u(z),0,0). \quad (4)$$

Using equation (4) in equation (2) we get,

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\left(\eta - \frac{\rho\beta^2 \bar{\mu}H_0}{2}\right)} \frac{\partial p}{\partial x}. \quad (5)$$

Solving equation (5) under the boundary conditions $u = 0, \text{ at } z = h, u = U \text{ at } z = 0$ we obtain,

$$u = \frac{1}{\left(\eta - \frac{\rho\beta^2 \bar{\mu}H_0}{2}\right)} \frac{\partial p}{\partial x} \left[\frac{z^2 - zh}{2} \right] - \frac{Uz}{h} + U. \quad (6)$$

Integrating the continuity equation we obtain

$$\frac{\partial}{\partial x} \int_0^h u dz + w_h - w_0 = 0, \quad (7)$$

where $w_h = -\lambda$ (squeeze velocity), and $w_0 = 0$ (the lower plate is impermeable).

Substituting equation (6) in equation (7) yields

$$\frac{d}{dx} \left[-\frac{h^3}{\left(1 - \frac{\rho\beta^2 \bar{\mu}H_0}{2\eta}\right)} \frac{dp}{dx} + 6\eta Uh \right] = 12\eta\lambda. \quad (8)$$

For rough surface using stochastic averaging method [9, 10, 11] equation (8) is transformed to

$$\frac{d}{dx} \left[\frac{-g(h)}{\left(1 - \frac{\rho\beta^2 \bar{\mu}H_0}{2\eta}\right)} \frac{dp}{dx} + 6\eta U g(h)^{\frac{1}{3}} \right] = 12\eta\lambda, \quad (9)$$

where

$$g(h) = h^3 + 3h^2\alpha + 3(\alpha^2 + \sigma^2)h + 3\sigma^2\alpha + \alpha^3 + \varepsilon, \alpha = E^*(h_s), \sigma^2 = E^*[(h_s - \alpha)^2], \varepsilon = E^*[(h_s - \alpha)^3],$$

$E^*(x)$ denotes the expected value given by

$$E^*(x) = \int_{-c}^c xf(h_s)dh_s,$$

here α is the mean, σ is the standard deviation, ε is the measure of the symmetry of random variable h_s .

Introducing the dimensionless parameters

$$\begin{aligned} \bar{x} &= \frac{x}{l}, \bar{h} = \frac{h}{h_1}, \bar{p} = \frac{h_1^2 p}{U\eta_l}, \bar{\beta} = \frac{\rho\beta^2\bar{\mu}H_0}{2\eta}, \\ D &= \frac{l\lambda}{Uh_1}, \bar{\sigma} = \frac{\sigma}{h_1}, \bar{\alpha} = \frac{\alpha}{h_1}, \bar{\varepsilon} = \frac{\varepsilon}{h_1^3}, \end{aligned} \quad (10)$$

then the equation (8) and equation (9) reduces to

$$\frac{d}{d\bar{x}} \left[-\frac{\bar{h}^3}{(1-\bar{\beta})} \frac{d\bar{p}}{d\bar{x}} + 6\bar{h} \right] = 12D, \quad (11)$$

$$\frac{d}{d\bar{x}} \left[-\frac{g(\bar{h})}{(1-\bar{\beta})} \frac{d\bar{p}}{d\bar{x}} + 6g(\bar{h})^{\frac{1}{3}} \right] = 12D. \quad (12)$$

The film thickness in non dimensional form is taken as

$$\bar{h} = \frac{a}{1 + \bar{x} \log a}. \quad (13)$$

Solving equation (11) and equation (14) under the boundary conditions

$$\bar{p}(1) = \bar{p}(0) = 0, \quad (14)$$

the dimensionless pressure \bar{p} can be obtained as

$$\bar{p} = \int_0^x \frac{F}{G} d\bar{x} - 12D \int_0^x \frac{\bar{x}}{G} d\bar{x} - Q \int_0^x \frac{1}{G} d\bar{x}, \quad (15)$$

$$Q = \frac{\int_0^1 \frac{F}{G} d\bar{x} - 12D \int_0^1 \frac{\bar{x}}{G} d\bar{x}}{\int_0^1 \frac{1}{G} d\bar{x}}, \quad (16)$$

for smooth surface

$$E = (1 - \bar{\beta}), G = \frac{\bar{h}^3}{E}, F = 6\bar{h},$$

for rough surface

$$E = (1 - \bar{\beta}), G = \frac{g(\bar{h})}{E}, F = 6g(\bar{h})^{\frac{1}{3}}.$$

The dimensionless load carrying capacity is given by

$$\bar{W} = \int_0^1 \bar{p} d\bar{x} = \int_0^1 \frac{F}{G} (1 - \bar{x}) d\bar{x} - 12D \int_0^1 \frac{\bar{x}}{G} (1 - \bar{x}) d\bar{x} - Q \int_0^1 \frac{1}{G} (1 - \bar{x}) d\bar{x}. \quad (17)$$

Using equation (17) load capacity is calculated numerically by MATHEMATICA and graphs are drawn varying different parameters.

4 Results and Discussions

The calculated values of load capacity, \bar{W} for various values of material constant, $\bar{\beta}$ with and without squeeze velocity for rough surface are shown in fig 2 and 3 respectively. It is observed in figure 2 that when there is a squeeze velocity the load capacity decreases as the curvature increases. where as in figure 3 when there is no squeeze velocity the load capacity increase for curvature upto $a = 1.8$ and then decreases when $a \geq 2$. The variation of \bar{W} with respect to variance $\bar{\alpha}$ are shown in fig 4 and we see that load capacity decreases for increase in $\bar{\alpha}$ up to 0.02 and then increases. The effect of standard deviation $\bar{\sigma}$ on load capacity \bar{W} are shown in fig 5 and 6 respectively. In figure 5 and 6 it is seen that the load capacity decreases for increase in $\bar{\sigma}$ up to 0.4 and then increases. The calculated values of load capacity, \bar{W} for various values of material constant, $\bar{\beta}$ when $\lambda \neq 0$ and $\lambda = 0$ for smooth surface are shown in fig 7 and 8 respectively. In figure 7 is seen that when there is no squeeze velocity the load capacity decreases for increase in curvature. Where as in figure 8 when there is a squeeze velocity the load capacity increase upto $a = 1.8$ and then decreases when $a \geq 2$.

5 Graphs

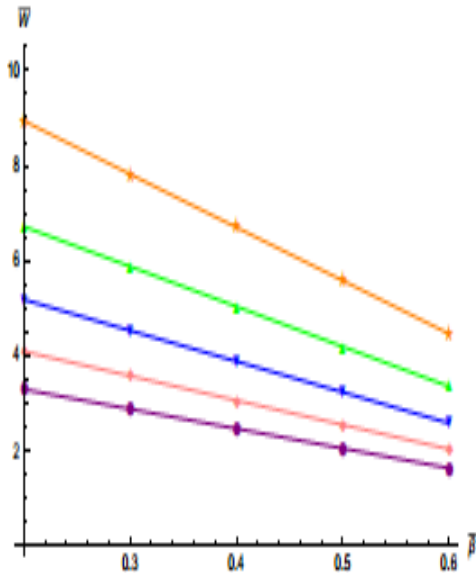


fig 2. \bar{W} versus $\bar{\beta}$ when $\lambda \neq 0$

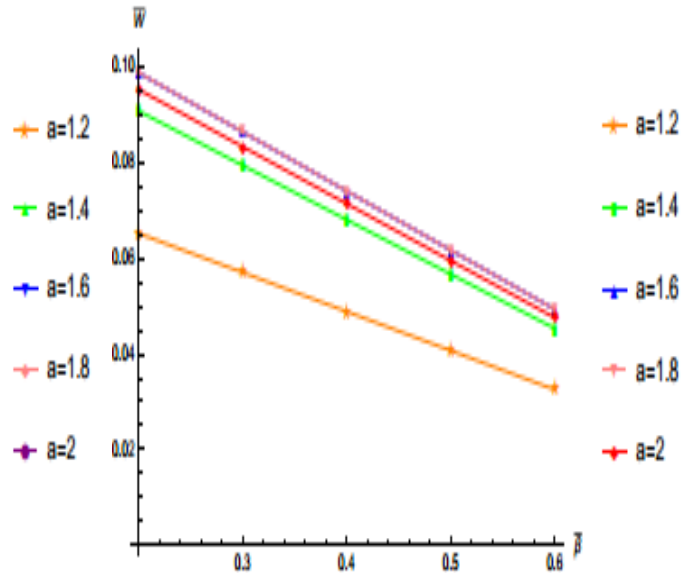


fig 3. \bar{W} versus $\bar{\beta}$ when $\lambda = 0$

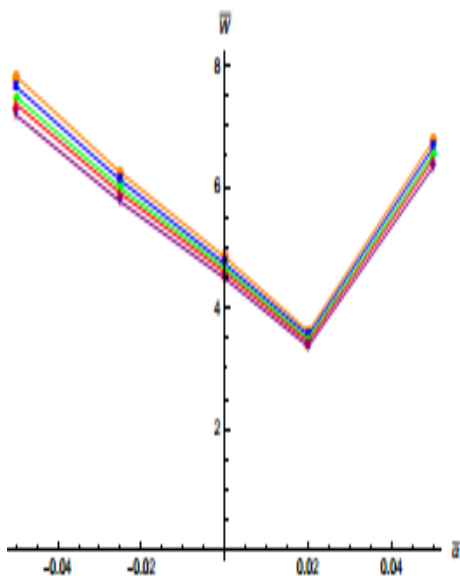


fig 4. \bar{W} versus $\bar{\alpha}$

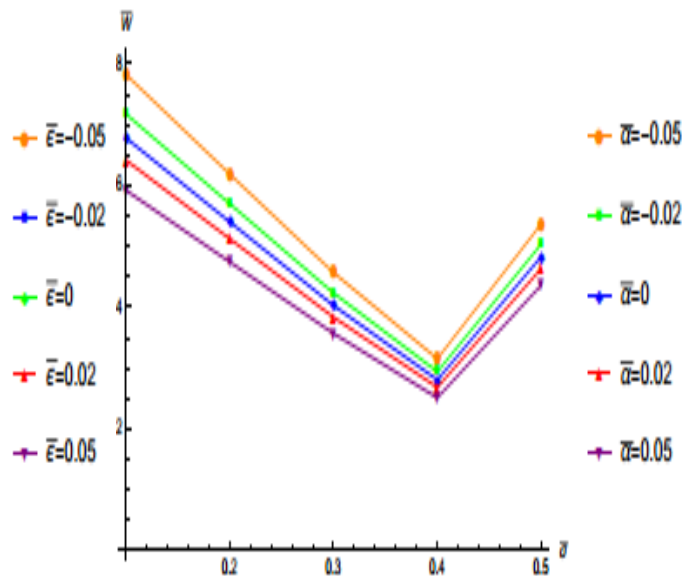


fig 5. \bar{W} versus $\bar{\sigma}$

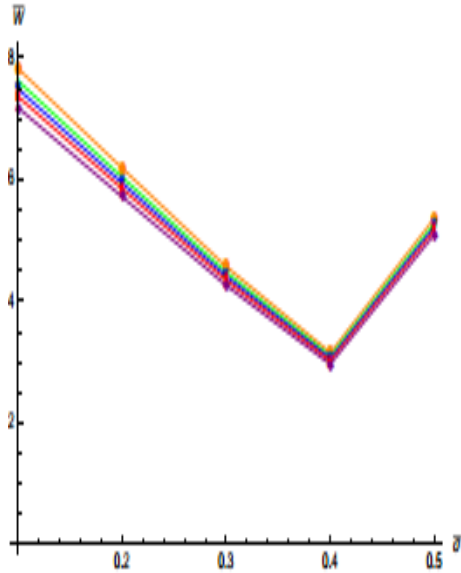


fig 6. \bar{W} versus $\bar{\sigma}$

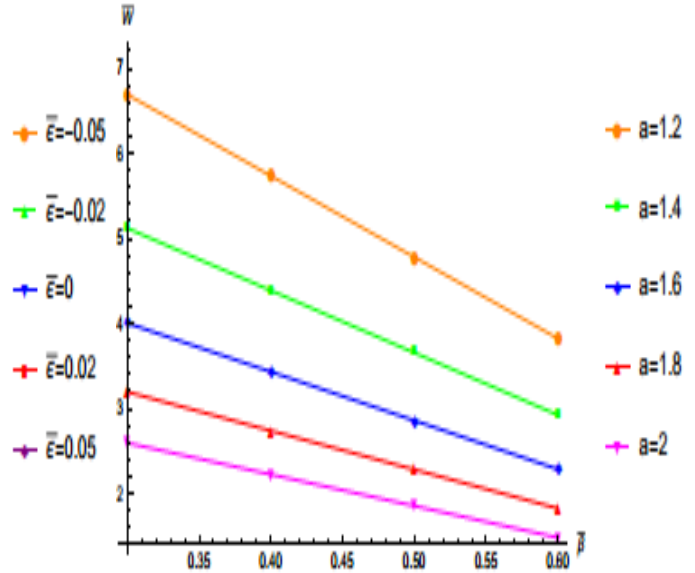


fig 7. \bar{W} versus $\bar{\beta}$ when $\lambda = 0$

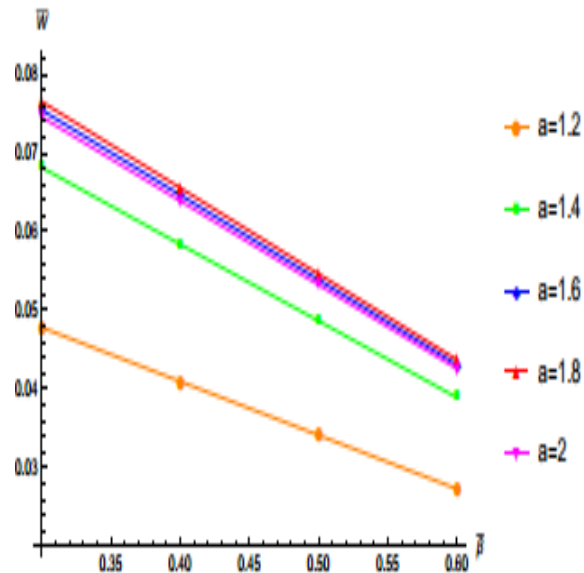


fig 8. \bar{W} versus $\bar{\beta}$ when $\lambda \neq 0$

References

- [1] V. K. Agarwal, Magnetic fluid based porous inclined slider bearing, *Wear* 107 (1986) 133-139.
- [2] Paras ram, P. D. S. Verma, Ferrofluid lubrication in porous inclined slider bearing, *Indian J. pure appl. Math.* 30(12) (1999) 1273-1281.
- [3] R. C. Shah, M. V. Bhat, Ferrofluid lubrication in porous inclined slider bearing with velocity slip, *Intl. J. Mech. Sci.* 44(2002) 2495-2502.
- [4] S. J. Patel, G. M. Deheri, J. R. Patel, Ferrofluid lubrication of a rough porous hyperbolic slider bearing with slip velocity, *Tri. Int.* 36(3) (2014) 259-268.
- [5] G. M. Deheri, J. R. Patel, N. D. Patel, Shliomis model based ferrofluid lubrication of a rough porous convex pad slider bearing, *Tri. Int.* 38(1) (2016) 57-65.
- [6] L. N. Achala, C. S. Asha, Study on ferrofluid bearings and their load capacity, *IJMCR*, 4(6) (2016), 1475-1480.

- [7] J. R. Patel, N. D. Patel, G. M. Deheri, Shliomis model based ferrofluid lubrication of a rough porous convex pad slider bearing, *Trib. Int.* 38 (1) (2016) 57-65.
- [8] N. D. Patel, G. M. Deheri, Hydromagnetic lubrication of a rough porous parabolic slider bearing with slip velocity, *Appl. Mech. Engg.* 3 (3) (2014).
- [9] H. Christensen, K. C. Tonder, Stochastic models of hydrodynamic lubrication, *Tribology of rough surfaces*, SINTEF. (1969) 10/69-18,.
- [10] H. Christensen, K. C. Tonder, Parametric study and comparison of lubrication models, *Tribology of rough surfaces*, SINTEF. (1969) 22/69-18.
- [11] H. Christensen, K. C. Tonder, The hydrodynamic lubrication of rough bearing surfaces of finite width, *ASME-ASLE lubrication conference*,(1970)70-Lub-7.
- [12] R. Aris, *Vectors, tensors, and the basic equations of fluid Mechanics*, Dover (1989).
- [13] R. E. Rosensweig, *Ferrohydrodynamics*, Dover (2014).