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GENERALIZED DERIVATIONS IN PRIME RINGS

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ABSTRACT

To study the derivations which satisfy certain differential identities on the general derivation of the prime ring. If a derivation exists as d: R→R where F (a, b) = F(a) b + a d(b)holds for all a, b ∈ R. Here in this paper, we have proved that V ⊆ Z (R) or d = 0, for all x ∈ I. Then, d(x) = λ [x, a], for all a ∈ Z. or x ∈ I.

KEYWORDS:

Generalized derivative, Abelian group, Prime ring, Derivation, commutative

INTRODUCTION

An algebraic structure of a rings are generalize field even multiplicative inverse and commutative need not exist. A ring is associated with two binary operations which satisfies the properties of addition and multiplication of an integers.

An element of a rings can be complex or integer numbers or non-numerical objects such as polynomials, functions, power series and square matrices.

R is a commutative ring with unity a belongs to R is called prime ring if,

- a is not equal to zero.
- a non-unit
- If a / bc in R then, a/b in R, a/c in R.

This property is called prime property

Prime ring is a derivation, then one of them is zero,

If d is a derivation of a. prime ring

i.e, for all elements of the ring

x y (x) – d(x)x is Central,

Then either d = (0) or the ring is commutative.

By definition we know that A ring R is called prime if and only if (xay) = 0,

Implies that x = 0 or y = 0.

Thus, R will be a ring with Centre Z.

Now Let x and y belong to R.

The commentator x y or y x will be denoted by [x, y].

Therefore, a ring is prime, if x (R) y = 0

implies x = 0 or y = 0.

Now, an additive mapping ∂: R → R is called a derivation.

If ∂ (a, b) = ∂(a) y + a ∂(b) for all a, b ∈ R.

The study of commutability of prime rings with derivation.

The current analysis states that there has been an ongoing interest concerning the relationship between the commutative ring and the existence of certain special types of derivations of R.

Bresar, defined the following symbol.

By derivation of a ring, there exists d: R→R

i.e, F (x, y) = F(x)y + x d(y) for all x, y ∈ R.

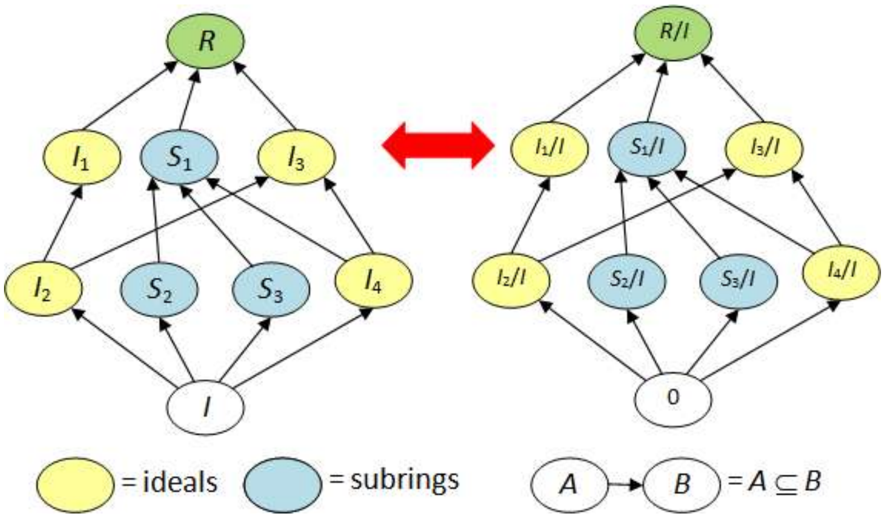
Here in this paper,

R will be a prime ring with Martindale ring of quotients as Qr(R),

which extended centroid C and central closure RC = RC,

Prime ring generalized that the ring derivation can be uniquely extended to a generalized ring derivation of Qr(R).

Albas, E. and Argac, N. showed that R is a non-commutative ring and a generalized ring derivation



Correspondence for the lattices of ideals and subrings between those

of *R* which contain I, and those of *R*/*I*.

In general, an idea can be contained in a subring, the ideal is the whole ring *R*

recollected that R is prime ring if a R b = 0

This implies, a = 0 or b = 0.

By an additive map of ring d: R→R is called a derivation

if d(x, y) = d(x)y + x d(y) for all x, y ∈R.

By Generalized derivation additive function of F: R→R.

If there exists a derivation d: R→R

Which holds F(x, y) = F(x)y + x d(y) for all x, y ∈R.

By analysis, we have the study of generalized derivation in the context of algebras on certain normed spaces.

As U is an additive subgroup of R then U is called an ideal of R

for all u an element of U and r element of R of the ring R.

Methodology

To prove theorems following known results which will be used considerably.

Theorem 2.1.1. If V is a Lie ideal of R such that V2 ∈V for all v ∈ V,

then 2 v u ∈ V for all v and u ∈V.

Proof. For all w, u and v ∈ V,

we have u v + v u = [(u + v) 2− v2 − u2]∈V

On the other hand,

foru v – v u ∈ V.

On adding both expressions,

we have 2 u v ∈ U for all u and v ∈ V

Theorem 2.1.2. If suppose V Z(R) is a Lie ideal of R,

then C R (V) = Z(R)

where R is whole ring.

Theorem 2.1.3. If suppose V is a Lie ideal of R,

then C R ([v, v]) = C R (V).

Theorem 2.1.4. If U ⊆ Z(R), then V Z(R) for Set U = {v ∈ V | d(v) ∈ V}.

Proof. Assume that for V ⊆ Z(R) and R is a ring

Because [v, v] ⊆ V and d([v, v]) ⊆ V,

Thus we have [v, v] ⊆ U ⊆ Z(R).

Hence C R ([v, v]) = R. by theorem 2.1.2 C R (V) = Z(R). where R is whole ring

But (V) belongs to the whole ring R. by theorem 2.1.3, C R ([v, v]) = C R (V).

That is, R = Z(R) is a contradiction.

Theorem 2.1.5. If V Z(R) is a Lie ideal of R also, if x U y = 0, then x = 0 or y = 0.

Theorem 2.1.6. A group G of ring R cannot be a union of two of its proper subgroups of a

group G. If V ⊆Z(R), So U ⊆Z(R) of group G.

Proof: By theorem 2.1.4. for all x and y ∈ V we have d(x) * F(y) = 0.

By theorem 2.1.1 on Changing 2yz and by taking R=2,

we obtained d(x) ◦ F(y z) = 0 for all x, y and z∈ V

$[d(a) \circ b]d(z) - b\,[d(a),d(z)] + [d(a) \circ F(b)]\,z - F(y)\,[d(a), z] = 0$

By definition of field of ring R it shows that

$[d(a) \circ b]\,d(z) - b\,[d(a), d(z)] - F(b)\,[d(a), z] = 0.$

Now by replacing z by d(a)

$[d(a) \circ b]\,d^2(a) - b\,d(a), d^2(a) = 0$ for $a \in V$

Next, Replacing y by 2zy

$[d(x) \circ (z\,y)]\,d^2(x) - z\,y\,[d(x), d^2(x)] = 0$ for all x, y and $z \in V$

This implies that

$z\,d(x) \circ y\,d^2(x)\,d(x), z\,y\,d^2(x) - z\,y\,d(x), d^2(x) = 0$

Now on solving the above equations,

we obtained

$[d(x), z]\,y\,d^2(x) = 0$ for $x \in U$ and $y \in V$ and $z \in V$

Here in this case,

$[d(x), x]\,y\,d^2(x) = 0,$

Therefore,

either $[d(x), x] = 0$ or $d^2(x) = 0$ of ring R

Now let us suppose $V_1 = \{x \in V\ [d(x), x] = 0\}$

And $V_2 = \{x \in U\ d^2(x) = 0\}$.

Therefore,

Additive subgroups of U are $U_1$and U_2

Also, $U_1 \cup U_2 = U$.

Thus, either $U = U_1$ or $U = U_2$of group G by theorem 2.16.

Now by taking $U = U_1$, then by Theorem 2.1.6 we have $V \subseteq Z(R)$

On the other hand, if we take $U = U_2$ of a prime ring R

The following known results will be used considerably to prove theorems.

*Theorem 2.1.7.*Let us suppose V be a Lie ideal of R in such a way that V2 ∈V for all v an element of V,

then 2 v u ∈ V for all v and u an element of V.

*Proof.*As we have u v + v u = (u + v) 2 − v2−u2 ∈V for all w, u and v ∈V

On the other hand, u v − v u ∈V.

Now on adding both expressions, we have 2 u v ∈ U for all u and v ∈ V,

Thus, by theorem 2.1.2. Now let us supposeV Z (R) be a Lie ideal of R,

then we get C R (V) = Z (R)

where R is the whole ring.

Now again, by theorem 2.1.3. as V is a Lie ideal of R,

then CR([V, V])=CR(V) and by theorem 2.1.4. Set U = {v ∈ V |d(v)∈v} if U Z(R), then V⊆ Z(R).

By commutativity sigma prime ring R.

A generalized derivation F satisfies

Either d(x) *F(y) = 0 or [d(x), F(y)] = 0 or d(x) *F(y) x *y = 0 or [d(x) ◦ F(y)][x, y] = 0 or (d(x) *F(y))[x,y] = 0 and

[d(x) *F(y)] x *y = 0

For all x, y in an appropriate subset of R,

where R is a 2-torsion prime ring, U a non-zero sigma lie ideal.

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