International Research Journal of Mathematics, Engineering and IT
ISSN: (2349-0322)
Association of Academic Researchers and Faculties (AARF)
Impact Factor- 7.132 Volume 10, Issue 03, March 2023
Website- www.aarf.asia, Email : editoraarf@gmail.com

A STUDY OF RANK OF A COVARIANCE MATRIXAND ITS APPLUCATIONS

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Abstract: The purpose of this section is to provide an overview of the research that we have been conducting in our investigation into the direct change idea, with a focus on learning difficulties, common mental models that understudies may have about it, a system of an inherited rot that depicts an expected way by which this idea can be created, issues that understudies may comprehend as to registers of depiction, and the work that unique estimation conditions may plough. In this paper, the estimation of covariance matrices based on multivariate sign and rank vectors is discussed. Equivariance and robustness properties of the sign and rank covariance matrices are described. We show their use for the principal components analysis (PCA) problem. Limiting efficiencies of the estimation procedures for PCA are compared.

Keywords: Matrix, PCA

1. Introduction:

The RMP is used in a variety of ways in this section. An emphasis on the problem's universality and links across applications in many fields is one goal of the presentation, as is conveying the practical significance of the rank. Here are a few instances of covariance matrices with respect to their rank. Second-order statistics for random processes are utilised in statistics, econometrics, signal processing, and other domains where these difficulties emerge. Principal Component Analysis and Factor Analysis are two examples of second-order statistical data analysis processes that may be used to deal with a large amount of noisy data. Because of the noise, the covariance matrices that have been constructed have full rank (with probability one). The discovery of a low-rank covariance matrix is a natural consequence of these strategies. Because of their straightforward nature, low-rank covariance matrices are straightforward to comprehend and model. To demonstrate what I mean, consider the following constrained factor analysis problem:

$$\text{minimize Rank } (\Sigma)$$
$$\text{subject to } \|\Sigma - \Sigma\|_F \leq \epsilon,$$
$$\Sigma \geq 0$$
$$\Sigma \in C,$$

where$\Sigma \in R^{n \times m}$ is the optimization variable, Σ is the measured covariance matrix, C is a convex set denoting the prior information or assumptions on Σ, $\|\cdot\|_F$end le denotes the Frobenius norm of a matrix (other matrix norms can be handled as well). The constraint $\|\Sigma - \Sigma\|_F \leq \epsilon$ means that the error, $i.e.$, the difference between and the measured covariance in Frobenius norm, must be less than a given tolerance ϵ The constraint $\Sigma \geq 0$ ensures that we obtain a valid covariance matrix.

In the statistics terminology, the objective function, **Rank** Σ corresponds to the mimber of factors that explain Σ.If $C = R^{n \times m}$ ($i.e.$, no prior information), this See Section 4.1 for an SVD-based analytical solution to this issue. This is compounded by additional limitations, such as upper and lower boundaries on the entries of Σ.

2. Sensor array processing

In sensor array processing, data containing the superposition of a number of signals, corrupted by additive noise, is measured as papataly separated sensers. The vector of observations or measurements $y(t) \in R^P$ can be modeled as

$$y(t) \sum_{i=1}^{k} x_i\,(t) a_i + v(t),$$

where$x_i(t) \in R$ is the ith signal, vit) € R is the noise, and a $a_i R^P$ is a function of some signal-dependent characteristic of the sensor array's reaction to the ith signal Equivalently,

$$y(t) = Ax(t) + v(t),$$

where$x(t) = x_1(t), \dots, x_k(t)^T$and $A = [a_{i\ \dots}\, a_k]$. Each vector $y(t_i)$is a snapshot across the array of sensors at timet. G ven observations $y(t_i), \dots, y(t_N)$,it is desired to estimate the unknown number of signals, k where $k < p$

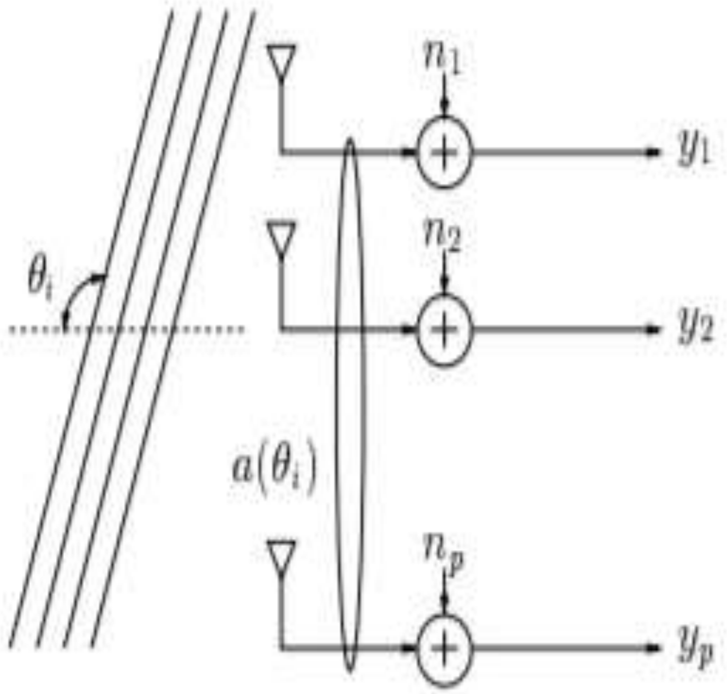

Figure 1:A general antenna array processing system

We assume that $x(t)$ has covariance matrix $\Sigma_{x,}$ and v (t) is white Gaussian noise, independent of $x(t)$with covariance matrix $\sigma^2 I$.The covariance of $x(t)$is given by $\Sigma_y = \psi + \sigma^2 I$, where $\psi = A\Sigma_x A^T$. we assume A to have full rank, we have $k =$ Rank $\Sigma_x =$ Rank (A$\Sigma_x A^T$) = Rank ψ Thus, the number of simals is expressed as the rank of a covariance matrix.

Antenna arrays, harmonic retrieval, and a slew of other applications run into this issue. Figure 1, for example, depicts how an antenna array processing system is set up. The complex amplitude of a plane-wave signal striking the array at an angle of 1 is represented here by θ(i.). each of the columns in A, which are denoted by the letter "a" ($\theta_{i.}$) in the figure, gives the response of the antenna array in the direccion$\theta_{i.}$

We now formulate the problem of estimating the number of signals k as well as the covariances $\Sigma_y, \psi,$ and σ^2.One constraint for Σ_y is to be consistent withthe observations, $e.g.$, to maximize the likelihood of observing $y(t_i), \ldots y(t_n)$, or to increase the probability over a certain level. Take the logarithm of the combined Gaussian distribution $f(y(t), \ldots, y(t_n)$,we obtain the log-likelihood function as

$$L(\Sigma_y) = -\frac{N}{2} log\ dat\Sigma_y - \frac{N}{2} Tr(\Sigma_y)^{-1}\Sigma_y - \frac{Np}{2}\log(2\pi),$$

where$\Sigma_{y=}(1/N)\sum_{i=1}^{N} y\,(t_i)y(t_i)^T$ i There is a connection between rank-constrained RMPs and RMPs with RMPs. If the number of signals is known to be less than or equal to k, V and op are approximated using the optimization problem

maximize$L(\psi + \sigma^2 I$

subject to **Rank**$\psi \geq K,$

$$\psi \geq 0$$

with variables V and o?. The solution to this problem is known to be

$$\psi_{opt} = \sum_{i=1}^{k}[\lambda_i(\Sigma) - \sigma^2]v_i(\Sigma)v_i(\Sigma)^T, \sigma_{opt}^2 = \frac{1}{p-k}\sum_{i=k+1}^{k}\lambda_i(\Sigma),$$

$\lambda_i$where I and v_i The eigenvalues and eigenvectors are denoted. Keep in mind that the above-mentioned technique also addresses the issue of ranks being confined

minimize $\|\Sigma_y - \psi - \sigma^2 I\|_F.$

subject to Rank $\psi \leq k$

$$\psi \geq 0,$$

such that the error between and is kept to a minimum We demonstrate in Chapter 4 how to solve this rank-constrained issue using SVD, which gives us the same result that we saw earlier. As a result, the final goal will be

$$\sum_{i=k+1}^{p}[\lambda_i\,(\Sigma) - \sigma^2]v_i(\Sigma)v_i(\Sigma)^T \| \sum_{i=k+1}^{p}(\lambda_i\,(\Sigma) - \sigma^2)^2,$$

which attains its minimum value if oº is chosen as the opt given in, the problem of estimating the number of signals is formulated as

minimize$-log\ dat\Sigma_y \boldsymbol{Tr}(\Sigma_y)^{-1}\Sigma_y + g(k)$

subject to $\Sigma_y - \sigma^2 I \geq 0$

where$g(k)$is a measure of 'complexity' of the model as a function of the number of free parameters in the model $(i.e., in\ \Sigma_y)$, which is in turn a function of k =Rank$(\ \Sigma_y -$ $\sigma 2I$=**Rank** ψ.

AIC [1] and Rissanen's Minimum Description Length (MDL) are two popular information-theoretic metrics for determining g(k). These are the two most prevalent options for g(k). Both conditions lead to quadratic functions of rank k in this problem: We now have the following: AIC(k) = $k(2p-k$ and MDL$(k) = \frac{1}{2}(logN)k2p-k)$.(Note that these problems, although related to the RMP are not RMPs themselves.)

Note that if k is fixed, the problem reduces to (4.5), whose optimal objective value in terms of k can be found using (26) To find the optimal k in (4.7), we simply need to check the value of this objective for $k = 1, \dots, p$(see [97] and references therein).

This is an example of an RMP that can be solved analytically, but there may be additional restrictions that make this impossible. Some of the variations in our data may have upper and lower boundaries y_i or know that, for example, y_i and $y_i$have a higher correlation than $y_i$and y_iWith such additional constraints, the resulting RMP is computationally hard.

3. Conclusion

This is an example of an RMP that can be solved analytically, but there may be additional restrictions that make this impossible. Some of the variations in our data may have upper and lower boundaries y_i or know that, for example, y_i and y_ihave a higher correlation than y_iandy_iWith such additional constraint, the resulting RMP is computationally hard.In this paper, the estimation of covariance matrices based on multivariate sign and rank vectors is discussed. Equivariance and robustness properties of the sign and rank covariance matrices are described. We show their use for the principal components analysis (PCA) problem. Limiting efficiencies of the estimation procedures for PCA are compared.

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