International Research Journal of Mathematics, Engineering and IT
ISSN: (2349-0322)
Association of Academic Researchers and Faculties (AARF)
Impact Factor- 5.489 Volume 7, Issue 2, February 2020
Website- www.aarf.asia, **Email** : editor@aarf.asia , editoraarf@gmail.com

Study on fractional integro-differential equations

Dr. Sudesh Rathee, Associate professor,
G.C.Bahadurgarh, Distt. Jhajjar, Haryana

Abstract:

Using fixed point theorems, the primary objective of this study is to provide existence results and stability conditions for a class of fractional order differential equations . Existence findings are derived from Schauder's fixed point theorem and the Banach contraction principle. In addition, the use of Krasnoselskii's fixed point theorem to develop stability conditions for a particular class of fractional order differential equations is given a lot of attention. The usefulness of the stability result is shown via the use of an example. Through using the characteristics of -distance mappings and -admissible mappings, we present the idea of generalized contraction mappings and show the existence of a fixed point theorem for such mappings. This is accomplished by mapping properties. In addition, we extend our conclusion to the theorems of coincidence point and common fixed point in metric spaces. Further, the fixed point theorems that are endowed with an arbitrary binary relation may also be deduced from our conclusions thanks to this line of reasoning.

Keywords: *principle, mappings, equations*

INTRODUCTION

It is well known that many problems in many branches of mathematics can be transformed to a fixed point problem of the form $Tx = x$ for self-mapping T defined on framework of metric space (X, d). In 1992, Banach introduced the concept of contraction mapping and proved the fixed point theorem for such mapping, which is called the Banach contraction principle, which opened an avenue for further development of analysis in this field. Several mathematicians used different conditions on self-mappings and proved several fixed point theorems in metric spaces and other spaces.

In 1969, Nadler established the fixed point theorem for multivalued contraction mapping by using the concept of Hausdorff metric which in turn is a generalization of the classical Banach contraction principle. Afterward, Kaneko extended the corresponding results of Nadler to single valued mapping and multivalued mapping which is also generalization of the result of Jungck . Subsequently, there are a number of results that extend this result in many different directions

FIXED POINT THEOREMS

Fixed point theorems concern maps f of a set X into itself that, under certain conditions, admit a *fixed point*, that is, a point x X such that $f(x) = x$. The knowledge of the existence of fixed points has relevant applications in many branches of analysis and topology. Let us show for instance the following simple but indicative example.

Example Suppose we are given a system of n equations in n unknowns of the form $g_j(x_1, \ldots, x_n) = 0$, $j = 1, \ldots, n$ where the g_j are continuous real-valued functions of the real variables x_j. Let $h_j(x_1, \ldots, x_n) = g_j(x_1, \ldots, x_n) + x_j$, and for any point $x = (x_1, \ldots, x_n)$ define $h(x) = (h_1(x), \ldots, h_n(x))$. Assume now that h has a fixed point $\bar{x} \in \mathbb{R}^n$. Then it is easily seen that $\bar{x}$ is a solution to the system of equations.

The Riesz mean ergodic theorem

If T is a non-expansive linear map of a uniformly convex Banach space, then *all* the fixed points of T are recovered by means of a limit procedure.

Projections Let X be a linear space. A linear operator $P : X \to X$ is called a *projection* in X if $P^2x = PPx = Px$ for every $x \in X$. It is easy to check that P is the identity operator on ran(P), and the relations $\ker(P) = \operatorname{ran}(I - P)$, $\operatorname{ran}(P) = \ker(I - P)$ and $\ker(P) \cap \operatorname{ran}(P) = \{0\}$ hold. Moreover every element $x \in X$ admits a unique decomposition $x = y + z$ with $y \in \ker(P)$ and $z \in \operatorname{ran}(P)$.

Proposition *If X is a Banach space, then a projection P is continuous if and only if* $X = \ker(P) \oplus \operatorname{ran}(P)$.

The notation $X = A \oplus B$ is used to mean that A and B are closed subspaces of

X such that $A \cap B = \{0\}$ and $A + B = X$.

proof If P is continuous, so is $I - P$. Hence $\ker(P)$ and $\operatorname{ran}(P) = \ker(I - P)$ are closed. Conversely, let $x_n \to x$, and $Px_n \to y$. Since $\operatorname{ran}(P)$ is closed, $y \in \operatorname{ran}(P)$, and therefore $Py = y$. But $Px_n - x_n \in \ker(P)$, and $\ker(P)$ is closed. So we have $x - y \in \ker(P)$, which implies $Py = Px$. From the closed graph theorem, P is continuous. □

Theorem [F. Riesz] *Let X be a uniformly convex Banach space. Let*

$T : X \to X$ be a linear operator such that

$$\|Tx\| \leq \|x\|, \qquad \forall\, x \in X.$$

Then for every $x \in X$ the limit

$$p_x = \lim_{n\to\infty} \frac{x + Tx + \cdots + T^n x}{n+1}$$

exists. Moreover, the operator $P : X \to X$ defined by $Px = p_x$ is a continuous projection onto the linear space M = $\{y \in X : Ty = y\}$.

proof Fix $x \in X$, and set

$$C = \overline{\mathrm{co}(\{x, Tx, T^2x, T^3x, \ldots\})}.$$

STATEMENT OF THE PROBLEM

Because to the discovery of fuzzy set theory, we can now deal with a wide range of uncertain and real-world situations. When it comes to fuzzy set theory, every object is allocated a degree of membership ranging from zero to one. In classical metric space theory, it is difficult to deal with distance functions that have inexact values; as a result, the concept of fuzzy metric space was developed to deal with such cases. Nonlinear analysis is used to tackle nonlinear problems in a wide range of fields, including mathematics, physics, and industry. The field of nonlinear analysis known as fixed-point theory is included in this category. It is used to investigate the situations in which both single-valued and multivalued mappings provide answers to the problems. A range of topics in numerous areas of mathematics, including differential equations, optimization theory, and variation analysis, may be simulated using the equation.

$$u = Du$$

REVIEW OF LITERATURE

Brouwer (2012) was the first to prove a fixed point theorem which states that a continuous mapping of a closed unit ball in n-dimensional Euclidean space has atleast one fixed point. Several proofs of this basic result can be found in the existing literature. Alexendroff and Hopf (2013) proved Brouwer's theorem by using the tools from algebraic topology while Birkhoff and Kellogg and Dunford and Schwartz. used classical methods of analysis and determinant to prove the same theorem. Theorems confined to the subspaces of R" are not of much immediate use in functional analysis, where one is usually concerned with the case that E is infinite dimensional subset of some function space. Over a four decades ago Birkhoff and Kellogg were the first to obtain the first infinite dimensional fixed point theorem.

In fact, Brouwer's fixed point theorem was used by Birkhoff and Kellogg in 1922 in proving the existence theorems in the theory of differential equations. Afterwards Schaiider (2014) extended Brouwer's fixed point theorem to the case in which E is a compact convex subset of anormed space.

Later on, TychonofiF (2015) extended Schaiider's results from normed spaces to an arbitrary locally convex space. Banach obtained the fixed point theorem for contraction mappings which is very famous because its proof is simple and does not require much topological background.

In recent years Kannan (2016), Husain and Sehgal, Caristi etc. have considered several generalizations of contraction mappings and proved a multitude of results. Since then many generalizations of the Banach Contraction Theorem have appeared, Chu and Diaz and Bryant observed that for a continuous mapping T of a complete metric space into itself such that T" is a contraction mapping of X for some positive integer k, then T has a unique fixed point. Rakotch and Boyd -Wong have attempted to generalize the Banach Contraction Theorem by replacing the lipschitz constant k by some real valued function whose values lie in (2016).

Common Fixed Point Theorems on Metric Spaces

Wardowski (2012) introduced the concept of F-contraction, which is later generalized in the name of F-weak contraction by Wardowski and Dung (2014); Dung and Hang (2015), Hussain et al. (2015), Piri and Kumam (2016) are some others who studied and extended the theory further. The concept of Suzuki F-contraction is defined by Piri and Kumam (2014).

Sgroi and Vetro (2013) introduced the concept of closed multivalued F-contraction and proved some fixed point theorems using Hardy-Rogers type of multivalued F-contractions on complete metric spaces. Ahmad et al. (2015) proved some fixed point theorems using two new classes of control functions and a fixed point theorem of Suzuki-Hardy-Rogers type in the contexts of F-contraction and Multivalued generalized F-contraction respectively.

The existence of solution for a coupled system of fractional integro-differential equations using Schauder fixed point theorem is proved by Babakhani (2013). Anber et al. (2013) proved the unique existence of a solution, for a class of boundary value problems of nonlinear fractional differential equations, with integral boundary conditions, using Banach fixed point theorem. Zada et al. (2018) defined L-cyclic (α, β)s contraction in b-metric spaces and studied the existence of the unique solution for fractional differential equations.

Khojasteh et al. (2015) developed a new type of contraction called Z-contraction using the concept of simulation functions; Roldan Lopez-de Hierro et al. (2015) slightly modified the notion of simulation functions and proved some fixed point theorems. Ran and Reurings (2003) introduced the concept of partially ordered metric space and proved an analogue of Banach's fixed point theorem.

Bhaskar and Lakshmikantham (2006) proved a coupled fixed point theorem in a partially ordered metric space, using a mixed monotone property, which is later extended by Sintunavarat et al. (2012). Sabetghadam et al. (2009) extended the theory of Bhaskar et al. in the context of partially ordered cone metric space. Further it is extended by Luong et al. (2013) and Sedghi et al. (2014).

OBJECTIVES OF THE STUDY

1. To study on fuzzy sets and involved it as an instrument for managing vulnerability emerging out of absence of data about specific complex framework
2. Discuss common and connected fixed point theorems and propose the concept of modified intuitionist fuzzy metric space.
3. To study on Common fixed point theorems in the realm of G-metric space, which generalises various comparable results.

RESEARCH METHODOLOGY

The research would be organized according to the chapters. It is descriptive in nature. Nonlinear analysis is employed in a wide range of fields, including mathematics, physics, and industry. The descriptive research approach was utilized in the current study to gather information. It is necessary to gather data in order to test a hypothesis, which is accomplished using the Descriptive Research Design. It is also considered as a survey design in other instances.

Sampling design

Selection of Samples

Researcher As a result, this fundamental theorem has become an extremely popular method for Common Fixed Point Theorems in Generalized M - Fuzzy Metric Spaces, Fixed Point Theorems in Generalized M - Fuzzy Metric Spaces, and other related topics. The explanation and description will be provided in descriptive method to facilitate further research into the subject in greater depth.

Data Collection Procedures

Specifically, certain fixed point and fixed point theorem in generalised probabilistic and generalised M-fuzzy metric spaces are discussed.

Data collection strategy

The data used in this research was gathered from both primary and secondary sources, and the results were analyzed. In order to test the assumptions and to create the cover for the current research, a Data from primary and secondary sources were used in this mathematic investigation.

1 . Primary

The fixed point theorems are used to get the main data that is needed.

2. Secondary methods

The secondary data is gathered from a variety of sources, including generalized probabilistic and generalized fixed point theorems M -fuzzy metric spaces and other related resources.

DATA ANALYSIS

Some Fixed Point and Coincident Point Theorem in Generalized M-Fuzzy Metric Spaces

Theorem Let (X, M, *) stand for a generalised M-fuzzy metric space, and further assume that $T: X \to X$ X exist a mapping in such a way that for everyone $x \neq y \neq z \in X$ and $t > 0$.

$$\mathcal{M}(Tx, Ty, Tz, t) > \min\{\mathcal{M}(x, y, z, t), \mathcal{M}(x, Tx, Ty, t), \mathcal{M}(z, Ty, Tz, t)\}$$

if there is a point x0 ∈ X such that the sequence { Tn (x0)} has a subsequence that converges to u, then this statement is true. If this is the case, u is the only fixed point in T..

Proof:

Let $x_0 \in X$ be any non-deterministic element that is fixed in X.

If this is the case, then there is. x1 ∈X such that x_1 = Tx0

Similarly there exists x2 ∈ X such that x2 = Tx_1 = T2 x0

Keeping up this pattern will eventually result in a series. xn = T n x0 for all n ≥ 1 in X.

Suppose x_n = x_n+1 for some n.

Then x_n = Txn, Thus x_n = u is a fixed point of T.

Let's take it as a given that xn ≠ xn+1 for all n

For n ≥ 1, We have

M (xn, xn, xn+1, t) = M (Txn-1, Txn-1, Txn, t)

> min { M (xn-1, xn-1, xn, t), M (xn-1, Txn-1, Txn-1, t),

M (xn, Txn-1, Txn, t)}.

= min { M (xn-1, xn-1, xn, t), M (xn-1, xn, xn, t), M (xn, xn, xn+1, t)}.

Results on G-metric space by using CLRg property

The initial consequence is a generalised strict contractive condition, and it extends to. The first theorem of.

Theorem. [Note: Let f and g be two self mappings of a symmetric G-metric space (Y, G) that satisfy the CLRg property and assume that they are weakly compatible.

$$G(fy_1, fy_2, fy_3) < \max\Big\{G(gy_1, gy_2, gy_3), \frac{G(fy_1, gy_1, gy_1) + G(fy_2, gy_2, gy_2) + G(fy_3, gy_3, gy_3)}{3}, \frac{G(fy_2, gy_1, gy_1) + G(fy_3, gy_2, gy_2) + G(fy_1, gy_3, gy_3)}{3}\Big\},$$

∀ y1, y2, y3 ∈ Y . Then f and g share a single unique fixed point in common..

Proof. By the definition of CLRg property, ∃ a sequence $\{\alpha_n\}$ in Y such that $\lim_{n\to\infty} f\alpha_n = \lim_{n\to\infty} g\alpha_n = g\alpha$ for some α ∈ Y . Consider

$$G(f\alpha_n, f\alpha, f\alpha) < \max\Big\{G(g\alpha_n, g\alpha, g\alpha), \frac{G(f\alpha_n, g\alpha_n, g\alpha_n) + G(f\alpha, g\alpha, g\alpha) + G(f\alpha, g\alpha, g\alpha)}{3}, \frac{G(f\alpha, g\alpha_n, g\alpha_n) + G(f\alpha, g\alpha, g\alpha) + G(f\alpha_n, g\alpha, g\alpha)}{3}\Big\}.$$

On letting $n \to \infty$, we obtain G $(g\alpha, f\alpha, f\alpha) \leq \frac{2}{3}G(g\alpha, f\alpha, f\alpha)$ which implies gα = fα Therefore, the place where f and g meet is denoted by the symbol. Let r = fα = gα. By weak compatibility of (f, g), we have fr = fgα = gfα = gr.

To prove fr = r: Suppose fr 6= r, then

$$\begin{aligned} G(fr, r, r) &= G(fr, f\alpha, f\alpha) \\ &< \max\Big\{G(gr, g\alpha, g\alpha), \frac{G(fr, gr, gr) + G(f\alpha, g\alpha, g\alpha) + G(f\alpha, g\alpha, g\alpha)}{3}, \frac{G(f\alpha, gr, gr) + G(f\alpha, g\alpha, g\alpha) + G(fr, g\alpha, g\alpha)}{3}\Big\} \\ &< G(fr, r, r), \quad \text{a contradiction.} \end{aligned}$$

Therefore, the fixed point that both f and g share is r. It is not difficult to demonstrate that the fixed point is one of a kind.

We now illustrate this theorem by giving an example.

Example . Let Y = and G : Y × Y × Y → [0,∞) defined by

$$G(y_1, y_2, y_3) = \begin{cases} 0 & \text{if} \quad y_1 = y_2 = y_3 \\ \max\{y_1, y_2, y_3\} & \text{in all other cases .} \end{cases}$$

Then (Y, G) is a symmetric G-metric space. Define f, g : Y → Y by

$$fy = \begin{cases} 5 & \text{if } y \leq 5 \\ 3 & \text{if } y > 5 \end{cases} \quad \text{and} \quad gy = \begin{cases} \frac{y+5}{2} & \text{if } y \leq 5 \\ 10 & \text{if } y > 5 . \end{cases}$$

In this case, both f and g fulfil the requirements of the CLRg property. Consider this with reference to a series. $\{\alpha_n\} = \{5 - \frac{1}{n}\}$ for all n.

Then $f\alpha_n = f\left(5 - \frac{1}{n}\right) \to 5$ and $g\alpha_n = g\left(5 - \frac{1}{n}\right) = \frac{5 - \frac{1}{n} + 5}{2} \to 5.$

The Common Fixed Point Theorem and Its Properties Within Cone Metric Space, With Applications to Commutative Mapping and Orbitally Continuous Mapping

Definition. Under the assumption that E is an actual Banach space and that X is not an empty set. Take it for granted that the mapping satisfies

$$0 \leq d(x,y) \text{ for all } x,y \in X \qquad \text{...(1)}$$

$$d(x,y) = 0 \text{ if and only if } x = y \qquad \text{..(2)}$$

$$d(x,y) = d(y,x) \text{ for all } x,y \in X \qquad \text{......................................(3)}$$

$$d(x,y) \leq d(x,z) + d(y,z) \text{ for all } x,y,z \in X. \qquad \text{...(4)}$$

After then, the distance d is referred to as a cone metric on X, and the space defined by X in conjunction with the cone metric d is referred to as cone metric space (X, d).

An arrangement { x_n } in a cone metric space X merges to x if and provided that

$$d(x_n, x) \to 0 \text{ as } n \to \infty \qquad \text{...(5)}$$

Let a sequence { x_n} In a metric space shaped like a cone, X converges to x. If $\{x_n\}$ unites to y then x = y. That is breaking point of { x_n } is extraordinary

A sequence $\{x_n\}$ X is considered to be a Cauchy sequence if and only if it is a cone-shaped metric space. $c \in E$

with $0 << c$ there is N such that

$$d(x_n, x_m) << c \text{ for all } n, m > N. \qquad \text{..(6)}$$

A sequence $\{x_n\}$ in cone metric space X converges to x , then $\{x_n\}$ is called a Cauchy sequence.

Let (X, d) be a complete cone metric space and f and g be two self mappings on X . If

$$w = fx = gx \text{ for some } x \text{ in } X. \qquad \text{..(7)}$$

Then, at that point, x is known as a happenstance point f and g and w is known as a point of fortuitous event of f and g .

Common Fixed Point Theorem In Cone Rectangular Metric Space

Definition . allow X to be a nonempty set and E be a genuine Banach space. Assume the mapping

$d : X \times X \to E$ satisfies

$$0 \leq d(x, y), \forall x, y \in X. \qquad \text{..(1)}$$

$$d(x, y) = 0 \text{ if and only if } x = y. \qquad \text{..(2)}$$

$$d(x, y) = d(y, x) \ \forall x, y \in X. \qquad \text{...(3)}$$

$$d(x,y) \leq d(x,w) + d(w,z) + d(z,y), \forall x,y \in X \quad ....(4)$$

and for all distinct point $w,z \in X - \{x,y\}$ (rectangular property).

CONCLUSION

Our study findings were based on numerous generalisations in the area of "fuzzy metric spaces (FMS)" as well as various "fixed point" outcomes in these spaces. A "fixed point" of a transformation is a point that stays unchanged throughout the transformation. In several domains, "fixed point theory" is primarily employed to explain equilibrium. It is crucial in differential equations, integral equations, partial differential equations, operator equations, and functional equations that occur in several fields such as financial mathematics, stability theory, economics, game theory, best approximation, and dynamic programming. The prominent mathematician Zadeh was familiar with the beneficial concept of "fuzzy sets" (1965). Later, fuzzy logic became the most powerful instrument in a variety of technological domains, including artificial intelligence, computer science, control engineering, medical science, and robotics, among others. Fuzzy set theory is a mathematical breakthrough that allows us to solve a variety of uncertain and real-world situations.

REFERENCES

[1] S. Banach, “Sur les operations dans les ensembles abstraits ´ et leur applications aux equations int ´ egrales,” ´ Fundamenta Mathematicae, vol. 3, pp. 133–181, 1922.

[2] S. B. Nadler, “Multi-valued contraction mappings,” Pacific Journal of Mathematics, vol. 30, pp. 475–488, 1969.

[3] H. Kaneko, “Single-valued and multivalued f-contractions,” Bollettino dell’Unione Matematica Italiana, vol. 6, no. 4, pp. 29– 33, 1985.

[4] N. Hussain and A. Alotaibi, “Coupled coincidences for multivalued nonlinear contractions in partially ordered metric spaces,” Fixed PointTheory and Applications, vol. 2011, p. 81, 2011.

[5] P. Z. Daffer and H. Kaneko, “Multi-valued f-contractive mappings,” Bollettino dell’Unione Matematica Italiana A, vol. 8, no. 7, pp. 233–241, 1994.

[6] H. Kaneko and S. Sessa, “Fixed point theorems for compatible multi-valued and single-valued mappings,” International Journal of Mathematics and Mathematical Sciences, vol. 12, no. 2, pp. 257–262, 1989.

[7] Latif and I. Tweddle, “Some results on coincidence points,” Bulletin of the Australian Mathematical Society, vol. 59, no. 1, pp. 111–117, 1999.

[8] H. K. Pathak, “Fixed point theorems for weak compatible multi-valued and single-valued mappings,” Acta Mathematica Hungarica, vol. 67, no. 1-2, pp. 69–78, 1995.

[9] H. K. Pathak and M. S. Khan, “Fixed and coincidence points of hybrid mappings,” Archivum Mathematicum, vol. 38, no. 3, pp. 201–208, 2002.

[10] W. Sintunavarat and P. Kumam, “Coincidence and common fixed points for hybrid strict contractions without the weakly commuting condition,” Applied Mathematics Letters, vol. 22, no. 12, pp. 1877–1881, 2009.

[11] H. Brezis, Analyse fonctionnelle - Th´eorie et applications, Masson, Paris (1983)

[12] K. Deimling, Nonlinear functional analysis, Springer-Verlag, Berlin (1980)

[13] N. Dunford, J.T. Schwartz, Linear operators, Interscience Publishers, New York (pt.I 1958, pt.II 1963)

[14] R.E. Edwards, Functional analysis, Holt, Rinehart and Winston, Inc., New York (1965)

[15] L.C. Evans, Partial differential equations, Amer. Math. Soc., Providence (1998)

[16] V.I. Istrat¸escu ˘ , Fixed point theory, D. Reidel Publishing Co., Dordrecht (1981)

[17] J.L. Kelley, General Topology, Van Nostrand Co., Princeton (1955)

[18] W.S. Massey, A basic course in algebraic topology, Springer-Verlag, New York (1991)

[19] Pazy, Semigroups of linear operators and applications to partial differential equations, Springer-Verlag, New York (1983)

[20] F. Riesz, B. Sz-Nagy, Functional analysis, Frederick Ungar Publishing Co., New York (1955)

[21] H.L. Royden, Real analysis, 3rd ed., Macmillan Publishing Company, New York (1988)

[22] W. Rudin, Principles of mathematical analysis, 3rd ed., McGraw-Hill Book Company, New York (1976)

[23] W. Rudin, Real and complex analysis, 3rd ed., McGraw-Hill Book Company, New York (1986)

[24] W. Rudin, Functional analysis, McGraw-Hill Book Company, New York (1973)

[25] Aamri M. and Moutawakil D. E. 2002. Some New Common Fixed Point Theorems under Strict Contractive Conditions. Journal of Mathematical Analysis and Applications. 270 (1): 181 - 188.

[26] Abbas. M., Ali. K. M. and Radenovic. S. 2010. Common Coupled Fixed Point Theorems in Cone Metric Spaces for w-Compatible Mappings. Appl. Math. Comput. 217: 195 - 202.

[27] Ahmed. A. and Rhoades. B. E. 2001. Some Common Fixed Point Theorems for Compatible Mapping. Indian J. Pure and Appl. Math. 32: 1247-1254.

[28] Ahmed. M. A. and Nafadi. H. A. 2013. Common Fixed Point Theorems for Hybrid Pairs of Maps in Fuzzy Metric Spaces. Journal of the Egyptian Mathematical Society. Http://Dx.Doi.Org/10.1016/J.Joems. 10.006.

[29] Alaca. C., Turkoglu. D. and Yildiz. C. 2006. Fixed Points in Intuitionistic Fuzzy Metric Spaces. Chaos, Solitons & Fractals. 29: 1073 - 1078.

[30] Al - Thagafi., M. A. And Shahzad N. 2008. Generalized I -Nonexpansive Selfmaps and Invariant Approximations. Acta Math. Sinica. 24(5): 867 - 876.