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Comparison of Stochastic Volatility Jump Diffusion Model without Shot Noise With Heston Model

Jitendra Singh
Shivaji College, University of Delhi, India. Email: jitu.iitd@gmail.com

Abstract

One well-known issue with the standard Black-Scholes (BS) approach when attempting to simulate option pricing or asset returns is that it is impossible to duplicate the observed skews/smiles for the second case and the empirical features of asset returns for the first. Adding jumps or stochastic volatility to the underlying process is a popular solution to this issue. This paper studies the stochastic volatility jump diffusion(SVJD) model without shot noise(SN) and compare with Heston model. Further, it is reviewing their theoretical properties, and focusing on their ability to model asset returns by analyzing their statistical properties. The models are calibrated usingU.S. OIL FUND (ETF) (NYSEArca: USO) option prices. Finally, numerical illustration of SVJD models without SN are consistent with the real data in compare to Heston model.

Keywords: Implied Volatility(IV), Jump Diffusion(JD), Stochastic Volatility(SV), Shot Noise(SN)

1. Introduction

The simple diffusion mechanism that asset values follow is shown by Bachelier around 1900. His primary goal was to offer the European call option at a reasonable price. The call option buyer is entitled, but not obligated, to purchase the underlying asset from the option seller at a predetermined price (the strike price) on a specific day (the expiration date). Since the geometric Brownian motion was introduced in 1959 as a more sophisticated market model that does not allow for negative prices, the volatility of this random walk can be understood as its diffusion coefficient in 1973. The Black-Scholes (BS) option pricing approach (1973) is based on the straightforward premise that volatility is a constant.The simplest straightforward method within the BS theory creates an analogous measure for the underlying asset. This modification of the measure ensured a fair pricing for the option. The European call option price can be easily found by averaging the ultimate payout as determined by the martingale measure and applying the proper discount. Both the BS model and the geometric Brownian motion model failed to produce the desired outcomes following the 1987 catastrophe. To determine the implied volatilities of empirical option prices, a number of tests have been conducted on the data. These tests verified that the implied volatility(IV) is U-shaped function of the ratio betweenthe option priceand the price of the underlying asset price[2].

Empirical studies of stock markets have shown that asset returns share some common statistical properties that cannot be explained by a normal distribution and are referred to in the literature as stylized facts. As a result, numerous research projects by financial economists have been launched to enhance and adjust the BS model in order to account for some or all of the three empirical occurrences mentioned above.Famous models include, for example, (a) the variance model with constant elasticity of J. C. Cox[7] and S. A. Ross[8] in 1976; (b) the SV models of Hull and White[9] in 1987, Stein and Stein[10] in 1991 and Heston[11] in 1993; (c) the SVJD models of Bates[12] in 1996, and Scott[13] in 1997(d) the JD models of R.C. Merton[6] in 1976 and S. G. Kou[4] in 2002. Ball and Torous [14] demonstrated that Jumps have an empirically significant impact on option pricing . Recent results from empirical research suggest that discrete jump components and stochastic volatility are essential components of the system that generates data. To adjust for longer maturities and jumps to represent the pricing of options with shorter maturities, stochastic volatility is required. Stochastic volatility and jump-diffusion together would make up the most logical model of stock prices, as argued in Bates[12], Andersen, Benzoni and Lund[15]and Bakshi[16].

2. SVJD Model Without SN

Given a riskfree probability measure *M*, which is assumed to exist, and under this measure the asset price *S(t)*adheres a JD process with zeromean and conditional variance *V(t)* at the riskfree rate *r*[3].

$$dS(t) = S(t)\left(\left(r - \lambda\bar{J}\right)dt\right) + \sqrt{V(t)}\, dW_s(t)) + \sum_{i=1}^{dK(t)} S(T_i)J(U_i) \quad (1)$$

where $\bar{J}$ is the mean jump amplitude, while $U_i$denotes the i-th jumpamplitude mark and $T_i$represents the i-th jumptime. *K(t)*denotesthe counting jump process(Poisson) withintensityλ, and

$$dK(t) = \begin{cases} 1, & \text{with probability } \lambda dt \\ 0, & \text{with probability } 1 - \lambda dt \end{cases}$$

with

$$E\,[dK(t)] = \lambda dt.$$

and

$$Var\,[dK(t)] = \lambda dt.$$

If the jump-amplitude mark U's density is evenly distributed from [m, n], then U's probability density function may be found using

$$f_U(u) = \frac{1}{n-m}\begin{cases} 1, & m \le u \le n \\ 0, & else \end{cases}$$

where m<0<n. and the mean is $\mu_U = E[U] = \frac{n+m}{2}$ and variance$\sigma_U^2 = Var[U] = \frac{(n-m)^2}{12}$.

J(U) is the Poisson jump-amplitude such that

$$U = log\,(J(U) + 1)$$

Therefore,

$$J(U) + 1 = exp\,(U)$$

so,

$$E\,[J(U)+\,1] = \,E\,[\exp(U)]$$

$$\bar{J}\,=\,E\,[J(U)]\,=\,E\,[exp\,(U)] - 1$$

now to find $\bar{J}$, we need to find $E\,[exp\,(U)]$, therefore distribution function of $exp\,(U)$ is

$$P\,[exp(U)\,\leq\,x]\,=\,P\,[U\,\leq\,log(x)]$$

$$P\,[U\,\leq\,log(x)]\,=\,\frac{log(x)\,-\,m}{n-\,m}\begin{cases}1, & e^m\leq\,x\leq\,e^n\\ 0, & else.\end{cases}$$

Therefore, density function of $exp(U)$ is as follow

$$f_{\exp\,(U)}(x) = \frac{1}{x(n\,-\,m)}\begin{cases}1, & e^m\leq\,x\leq\,e^n\\ 0, & else.\end{cases}$$

Therefore,

$$E\,[exp(U)]\,=\,\int_{e^m}^{e^n}\frac{x}{y(n-m)}\,dy$$

$$E[\exp(U)] = \frac{e^n-e^m}{n-m}$$

hence,

$$\bar{J} = E[\exp(U)] - 1 = \frac{e^n-e^m}{n-m} - 1$$

The instantaneous volatility is given by a squareroot diffusion process with pure meanreversion as

$$dV(t) = k\big(\theta - V(t)\big)dt + \sigma\sqrt{V(t)}dW_v(t) \quad (2)$$

Where θ and σ are mean and volatility of instantaneous volatility *V(t)* and the variable $W_s(t)$ and $W_v(t)$ areBrownian motions(BM) for *S(t)* and *V(t)*, respectively, with $E[dW_i(t)] = 0$, $Var[dW_i(t)]\,=\,dt$, for *i*= *s* or *v*, and correlation$Corr\,[dW_s(t), dW_v(t)]\,=\,\rho$.

The log-uniform distribution was selected for a number of reasons. Firstly, since log-double-exponential distribution or the log-normal has exponentially short tails contrast with the thick and flat tails of financial market(long term). Next, the jumps are tiny and therefore not observable from the continuous diffusion fluctuations around the logdouble exponential and lognormal near-zero peak. Furthermore, an indefinite jump domain is impractical since leaps in actual financial markets should be restricted, and it creates unjustifiable constraints for portfolio optimization[3]. There are two main benefits to the square-root stochastic-volatility process (2). Firstly, systematic volatility risk can be accommodated by the model. Second, the procedure produces an analytically manageable way to price options without compromising accuracy or necessitating unfavorable limitations on parameter values[3].

3. Heston Model[5]:

When volatility is stochastic, there no mean reversion and jump components in (1), here volatility follows mean reversion and square process

$$dS(t) = \mu_s S(t)dt + \sqrt{V(t)}S(t)dWs(t)$$

$$dV(t) = k(\theta - V(t))\,dt + \sigma\sqrt{V(t)}dWv(t)$$

Where $k, \theta, \sigma > 0$are constant parameters.

4. Comparison between SVJD Model Without Shot Noise and Heston Model

In this section, we give graphical as well as numerical illustrations to show how the SVJD model without shot noise behavior is important in option pricing models. There are many ways of illustrative demonstration such as results comparison of proposed model with reality data, with standard models and with simulations. Here we present the results comparison of SVJD with standard models. Here, we consider a standard model as Heston model. We visualize the behavior of the two models, by seeing following figures.

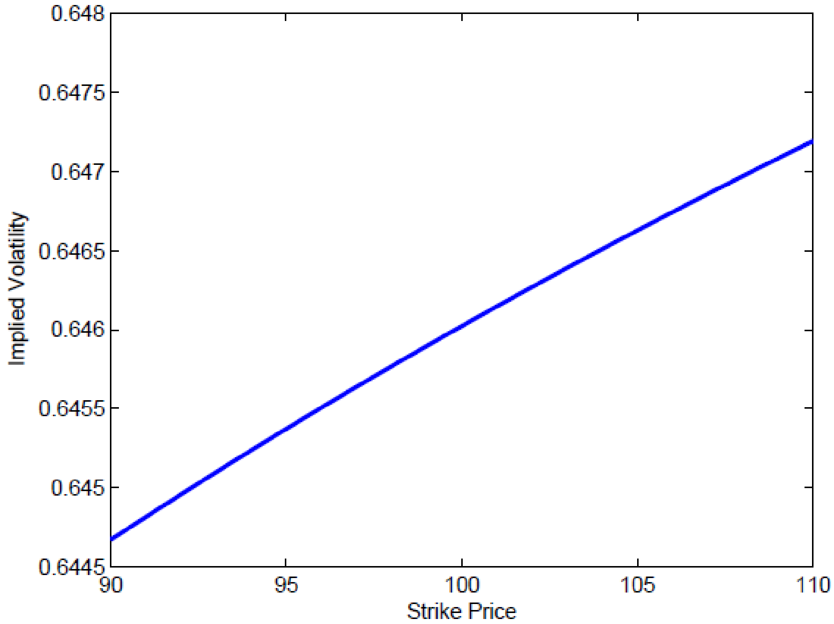

Figure 1: Volatility smile for Heston Model with current price(s0)=100, σ = .07, time=1 year,θ= 0.53, r=0.03, v=.012, $\rho = -0.622$

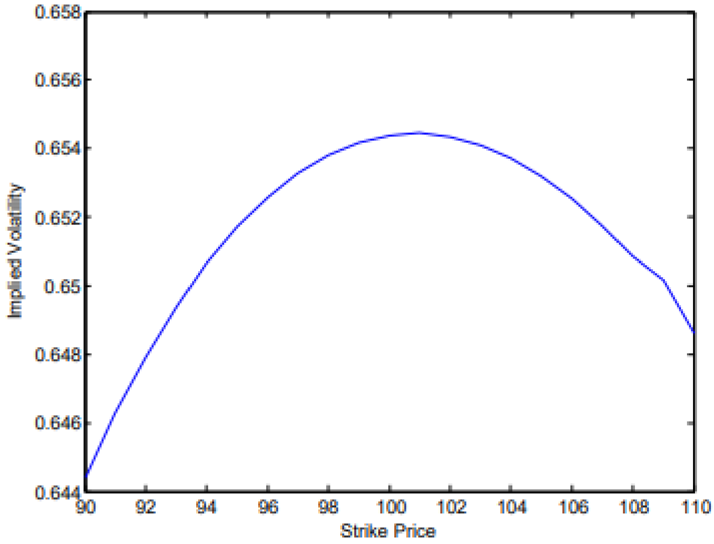

Figure 2: Volatility smile for SVJD without shot noise model with current price (s0) =100, , $\lambda = 64 \sigma = .07, time = 1 year$, $\theta = 0.53$, a=0.028, b=0.26, $r = 0.03$, $v = .012$, $\rho = -0.622$.

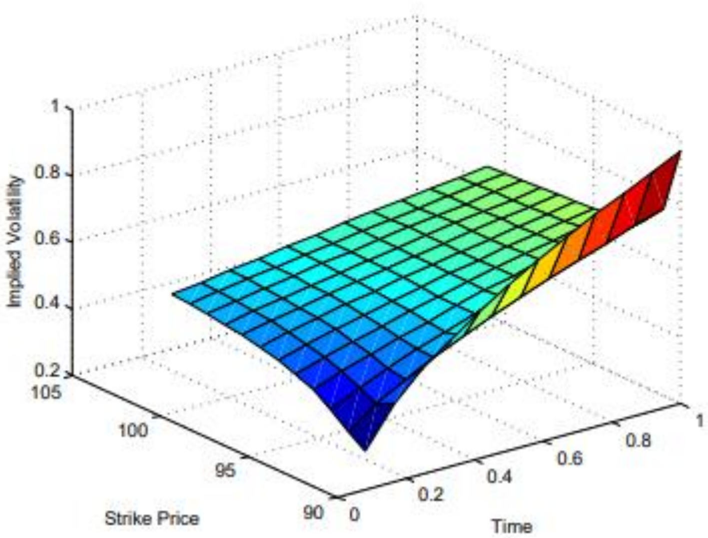

Figure 3: Implied Volatility surface for Heston Model with current price (s0) =100, $\sigma = .07$, $time = [1, 3]$ $year$, $\theta = 0.53$, $r = 0.03$, $v = .012$, $\rho = -0.622$

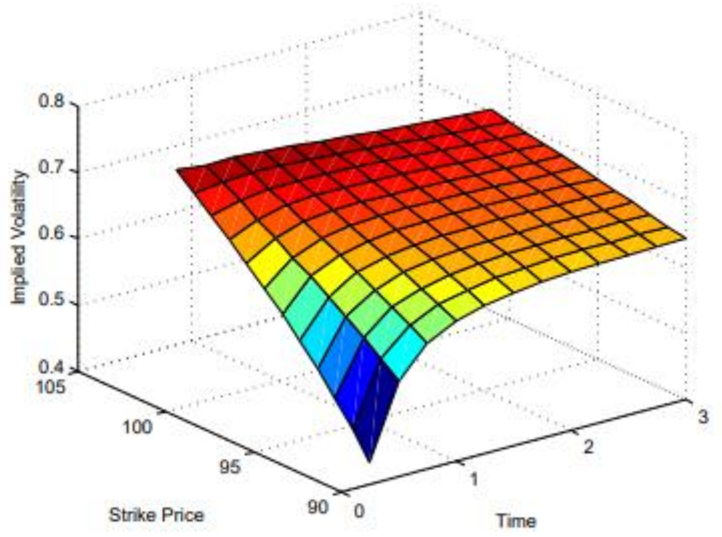

Figure 4: IV surface for SVJD without shot noise model with currentprice(s0)=100, $\lambda = 64$, $\sigma = .07$, $time = [1, 3] year$, $\theta = 0.53$, a=0.028, b=0.26, $r = 0.03$, $v = .012$, $\rho = -0.622$.

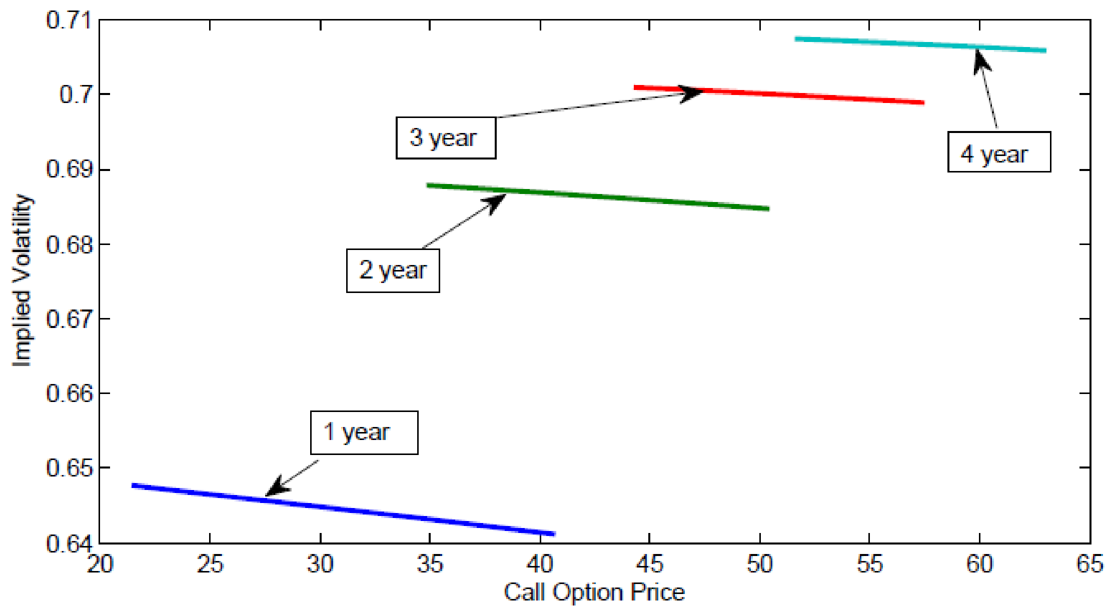

Figure5: Option Price vs IV for Heston Model with currentprice(s0)=100, $\sigma = .07$, $\theta = 0.53$, $a = -0.028$, $b = 0.026$, $r = 0.03$, $v = .012$, $\rho = -0.622$, for time to maturity as 1, 2, 3 and 4 years respectively.

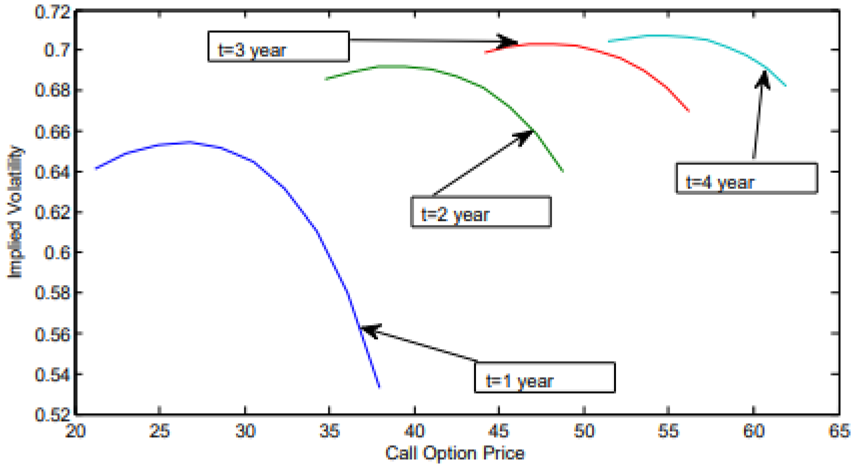

Figure 6: Option Price vs IV for SVJD without shot noise model withcurrent price(s0)=100, $\lambda = 64$, $\sigma = .07$, $\theta = 0.53$, $a = -0.028$, $b = 0.026$, $r = 0.03$, $v = .012$, $\rho = -0.622$, for time to maturity as 1, 2, 3 and 4 years respectively.

1. Figures 1 and 2 plot strike price against IV for the Heston model and the SVJD model without shot noise. TheIVis computed for varying strike prices. When the striking price exceeds the current price, the IV for the SVJD model without shot noise decreases, while for the Heston model it increases at the prior pace. These observations show that the IV for both models increases until the strike price is less than the current price.

2. Figures 3 and 4 depict the IV surfaces for the SVJD and Heston models, respectively, where the IVis computed for varying strike prices and times to maturity. For a brief period leading up to adulthood, it is seen that the smiles on both models are deeper, which is also in line with earlier research.

3. Figures 5 and 6, respectively, display the option price vs IV for the Heston Model and the SVJD model without shot noise. Keep in mind that the option price vs. volatility graph only illustrates how the option price behaves in a tumultuous market. In a stable market, a short-term shift in volatility results in a short-term change in the option price. But over time, the option price increases or improves despite little change in the market's turbulent circumstances.
It is shown that implied volatility increases for both models as maturity times grow, and that implied volatility decreases for a given maturity time when option prices rise.

5.Numerical Comparison of SVJD Model without shot noise and Heston Model

In this section, the European call option price for real data withHeston Model and SVJD Model without SN is compared .Theestimated parameter are S_0= 39.7,K = 35, λ = 38, σ = 0.0102, θ =0.2794, $a = -0.028$, b = 0.026, r = 0.0173, v = 0.0148, ρ = 0.3396, k = 3.56.

Maturity (Time)	**Use option price (Real Data)**	**Heston (Numerical)**	**SVJD without Shot Noise (Numerical)**
20/Nov/09	5.95	4.73	2.93
18/Dec/09	6.74	5.07	3.76
15/Jan/10	6.60	5.73	4.50
16/Apr/10	6.65	8.25	6.60
21/Jan/11	10.00	13.41	10.65
20/Jan/12	12.60	17.53	13.94

Table 1: Option price for different maturity date for USO real data,Heston model and SVJD model without shot noise.

6. Conclusion :

From the figures given in section 4 and table 1 in section 5, it can be seen that the SVJD model without SN is exhibited better result in comparison to Heston Model which is close to real data.

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