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THE ROLE OF FUZZIFICATION AND DEFUZZIFICATION IN TRAFFIC LIGHT CONTROLLER

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Abstract:

Today, the traffic light controller plays very important role in our daily life. Since traffic congestion is a problem based on uncertainty and ambiguity and fuzzy set theory deals uncertainty and ambiguity in the best sense, therefore we introduce the important part of fuzzy logic controller as "The Role of Fuzzification and Defuzzification in Traffic Light Controller."

Keywords: Boolean Logic, Fuzzy Logic, Linguistic Variables, Membership Functions, Triangular Membership Functions (TMF), Fuzzification, Defuzzification, Centroid Method.

1. Introduction:

A fuzzy set theory deals the truth based on uncertainty, ambiguity, and imprecision. At first, a fuzzy set is described by Lotfi A. Zadeh [1] in 1965 whose elements are of varying degrees of membership in [0, 1], where 0 is used for completely false and 1 is used for completely true. In 1975, Lotfi A. Zadeh also gave the concept of linguistic variables [2].He also explains the operator representations of linguistic hedges such as slightly, essentially, very, much, less, etc. [3]. He gave a procedure based on natural languages to calculate the probability distribution [4] and the theory of possibility based on fuzzy sets [5]. He [6] also explain the importance of fuzzy logic.

D. Dubois et al [7] describes the fundamental of fuzzy sets which provides the better result of reality than the binary representation of reality. A. De Luca et al [8] define a non-probabilistic entropy by using fuzzy set theory and C. Alsina et al [9] gave some logical and fuzzy connectives and their properties whereas G. DE Schrijver et al [10] define arithmetic operators based on interval - valued fuzzy sets. H. Bustince et al [12] obtained interval valued fuzzy sets from matrices. R. R. Yager [11] explains the extension principle based on interval - valued fuzzy sets and describes the concept of level sets.Thus; too many traffic light controllers were developed by taking different – different inputs to get the appropriate outputs [13], [14], [15]. Some traffic light controllers [16], [17] were developed for emergency vehicles.

2. **Fuzzy logic:**

A many valued logic having truth values in between completely false and completely true is called fuzzy logic. If completely false is denoted as '0' and completely true is denoted as '1', then truth value is any real number lying in [0, 1].

If the truth value is either 0 or 1, then the logic is said to be Boolean logic.

3. **Fuzzy Set and Membership Function:**

Let X be a universe of dissertation and let x be an element of X. Then, the fuzzy set $\tilde{A}$ is a collection of ordered pairs $\left(x, \mu_{\tilde{A}}(x)\right)$.

Thus; we can write it as;

$$\tilde{A} = \left\{\left(x, \mu_{\tilde{A}}(x)\right) : x \epsilon X\right\}$$

where $\mu_{\tilde{A}}(x)$ is a function from X to the interval [0, 1] and this function is known as membership function. It gives the degree of truth to which $x \epsilon \tilde{A}$.

The **membership function** of a fuzzy set $\tilde{A}$ (tilde underscore) is a function

$$\mu_{\tilde{A}} : X \rightarrow [0,1] \text{ such that}$$

$$\mu_{\tilde{A}}(x) = degreetowhichx\epsilon\tilde{A}.$$

Thus; It is the generalization of the characteristic function of crisp set and thelarger membership grades denote higher degrees of set membership.

<u>REPRESENTATION OF FUZZY SET:</u>

In 1965, Zadeh introduces two operators Σ and ∫ to represent a fuzzy set. The symbol Σ is used for the collection or aggregation of elements while the symbol ∫ is used for continuous function – theoretic aggregation.

(i) When X (universe of discourse) is finite and discrete, then the symbol Σ is used to represent the fuzzy set.

Thus; in this case, a fuzzy set can be represented as;

$$\tilde{A} = \left\{\frac{\mu_{\tilde{A}}(x_1)}{x_1} + \frac{\mu_{\tilde{A}}(x_2)}{x_2} + \cdots\cdots\right\} = \left\{\sum_i \frac{\mu_{\tilde{A}}(x_i)}{x_i}\right\}$$

here the horizonal bar (-) is a delimiter, not a quotient.

(ii) When X (universe of discourse) is continuous and infinite, then the symbol ∫ is used to represent the fuzzy set, i.e.,

$$\tilde{A} = \left\{\int \frac{\mu_{\tilde{A}} x}{x}\right\}$$

4. **Fuzzy Number and its Type:**

If a fuzzy set $\tilde{A}$ on real line satisfies the following properties –

(i) $h(\tilde{A}) = 1$, i.e., $\tilde{A}$ is normal.

(ii) $\alpha - cut\ of\ \tilde{A}$ is a closed interval; $\forall \alpha \in [0, 1]$, and

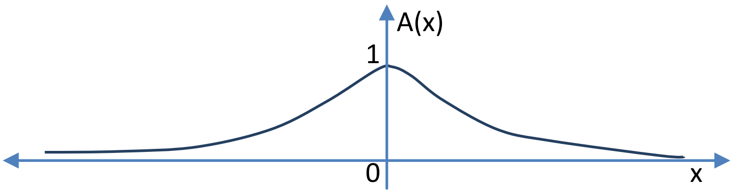
(iii) ${}^{0+}\tilde{A}$ (Support) is bounded.

Then, this fuzzy set is known as a fuzzy number.

QUASI FUZZY NUMBER:

A fuzzy number $\tilde{A}$ on R satisfying the following limit conditions is called a quasi-fuzzy number.

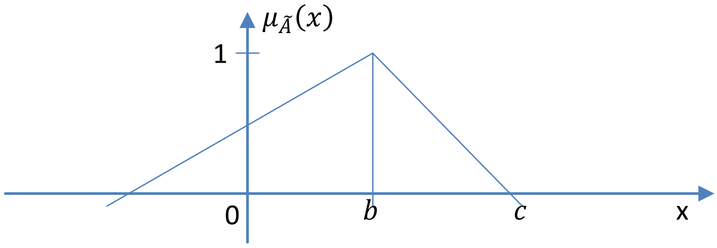
$$\lim_{x\to\infty} \mu_{\tilde{A}}(x) = 0, \lim_{x\to-\infty} \mu_{\tilde{A}}(x) = 0$$



TRIANGULAR FUZZY NUMBER:

The membership function of a triangular fuzzy number $\tilde{A}$ constituted with three points (a, b, c) is defined as;

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , \quad x < a \\ \dfrac{x-a}{b-a} & , \quad a \le x \le b \\ \dfrac{c-x}{c-b} & , \quad b \le x \le c \\ 0 & , \quad x > c \end{cases}$$



5. Linguistic Variables:

The variables whose states are fuzzy numbers representing linguistic concepts, such as very large, large, medium, small, and so on, as taken in a particular context, are called linguistic variables.

6. Fuzzification:

The process of converting a crisp quantity into a fuzzy quantity is known as fuzzification. Sometimes, fuzzification is used to convert a fuzzy set into a fuzzier set. By this operation, a crisp input value is translated into a linguistic variable.

Let $A = \left\{\frac{\mu_i}{x_i} : x_i \in X\right\}$ be a fuzzy set. Then we have to find fuzzified set $\tilde{A}$. For this, we use a common fuzzification (which is known as **support fuzzification** or, **S-fuzzification**) algorithm.

At first, we keep μ_i as constant and then transform x_i into a fuzzy set $P(x_i)$ representing the expression about x_i.

Now; the fuzzified set $\tilde{A}$ is given as;

$$\tilde{A} = \mu_1 P(x_1) + \mu_2 P(x_2) + \mu_3 P(x_3) + \cdots + \mu_n P(x_n)$$

where the symbol $\sim$ means fuzzified. Here $P(x_i)$ is the kernel of fuzzification.

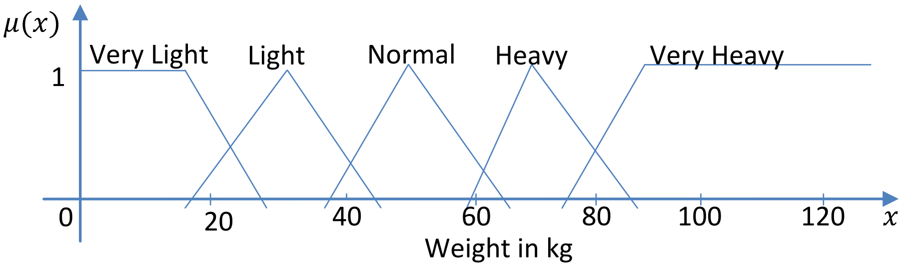
If we keep x_i as constant and express μ_i as a fuzzy set, then this process is known as **grade fuzzification** or, **g-fuzzification**.

There are many methods for assigning membership function. One of them is Intuition method, which is often used.

Intuition Method:

Human beings have the capacity based on their own intelligence and understandings, to develop membership functions. In this method, human beings generate membership functions using their common intelligence.

Let us consider the weights of people measured in kg in the universe, which are given as;

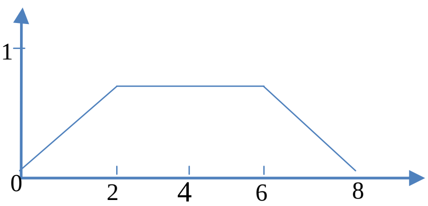


Here, each curve denotes the membership function corresponding tofuzzy variables (linguistic variables) very light, light, normal, heavy, and very heavy.

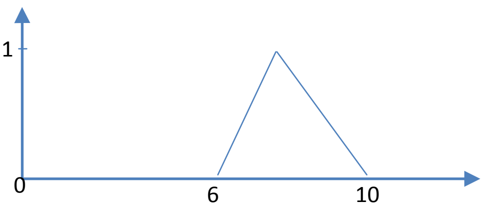
7. Defuzzification:

Defuzzification is the process of converting a fuzzy quantity into a precise quantity. Defuzzification is a decision – making algorithm which gives the best crisp results based on the fuzzy set. Thus; **defuzzification is the reverse process of fuzzification**.

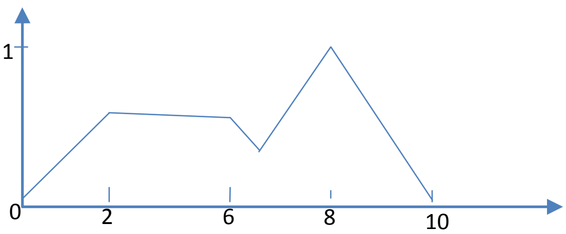
The output of two or more fuzzy membership functions is the logical union of those membership functions,which can be seen graphically as –



First membership function



Second membership function



Union of both parts

The purpose of defuzzification in artificial intelligence is to generate actionable outcomes into quantifiable values.

8. DEFUZZIFICATION METHODS:

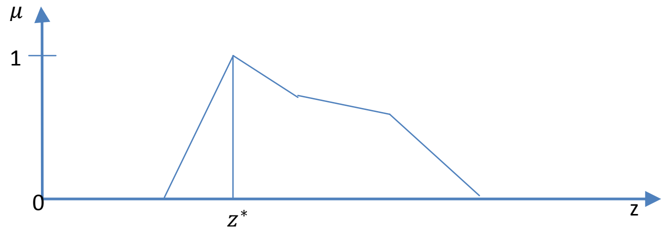
Here, the methods of defuzzification are explained one-by-one, which are given below –

(i) Max membership principle:

Let z^* be an element of Z such that it satisfies

$$\mu_{\tilde{A}}(z^*) \geq \mu_{\tilde{A}}(z); \forall\, z \in Z,$$

thenz^* is the de-fuzzified value. Sometimes, it is called the height method. It is considered as a peaked output function.

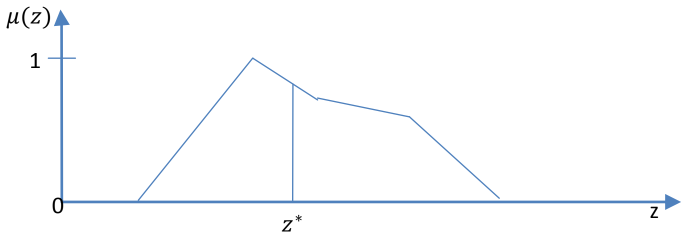


(ii) Centroid Method:

This method of defuzzification is the commonly used methodin the process of defuzzification. In this method, the de-fuzzified value z^* is definedas;

$$z^* = \frac{\int \mu_{\tilde{A}}(z) \cdot z\, dz}{\int \mu_{\tilde{A}}(z)\, dz}.$$

Here the sign □ is used for algebraic integration. Sometimes, it is called center of area or, center of mass or, center of gravity method.



Centroid defuzzification method

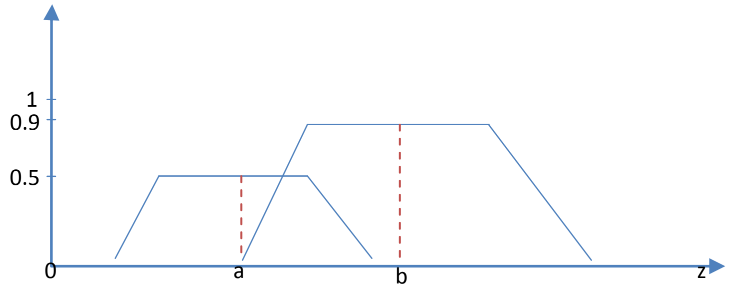
(iii) Weighted Average Method:

This weighted average method is applicable for symmetrical output function only and it is one of the more computationally efficient method. So, this is the mostused method in fuzzy applications. If $\overline{z}$ is the centroid of each symmetric membership function, then the de-fuzzified value z^* is definedas;

$$z^* = \frac{\sum \mu_{\tilde{A}}(\overline{z}) \cdot \overline{z}}{\sum \mu_{\tilde{A}}(\overline{z})},$$

Here the sign $\sum$ is used for algebraic sum.

Each membership function is weighted in the output by its respective maximum membership value.



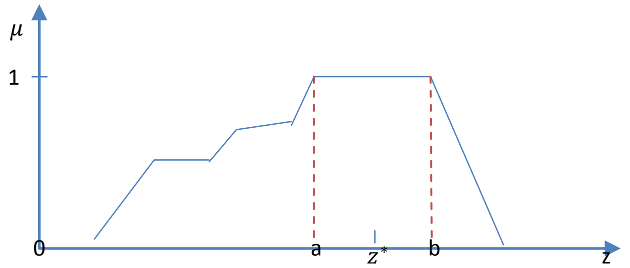
According to this figure, the de-fuzzified value $z^*$is given by;

$$z^* = \frac{a(0.5) + b(0.9)}{0.5 + 0.9}$$

(iv) <u>Mean – Max Membership:</u>

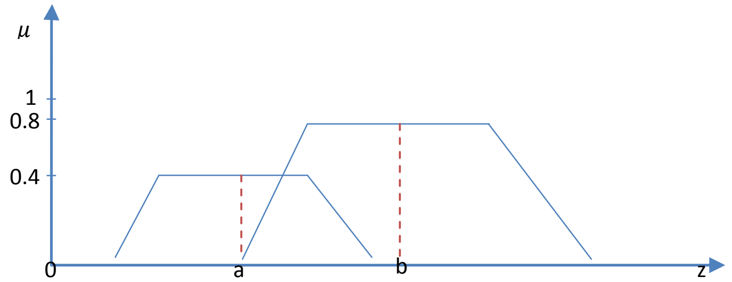
The Mean-Max Membership method is closely connected to max – membership method. Here locations of the maximum membership may be taken non-unique. The de-fuzzified value z^* is given by;

$$z^* = \frac{a + b}{2},$$



The output is also defined by

$$z^* = \frac{\sum_{i=1}^{n} \overline{z_i}}{n}$$



Two symmetrical membership functions

According to the above figure, the de-fuzzified value is given by

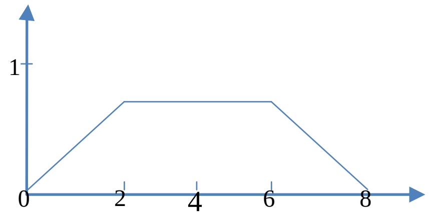
$$z^* = \frac{a + b}{2}.$$

Sometimes, this method is also known as Middle of Maxima method.

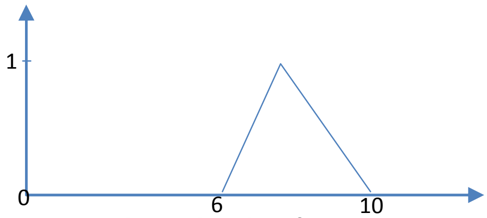
(v) Center of Sums:

In this method, wefind the algebraic sum of the individual fuzzy output subsets in the place of their union. If $\bar{z}$be the distance to the centroid of each of the respective membership functions, then the de-fuzzified value z^* is defined as;

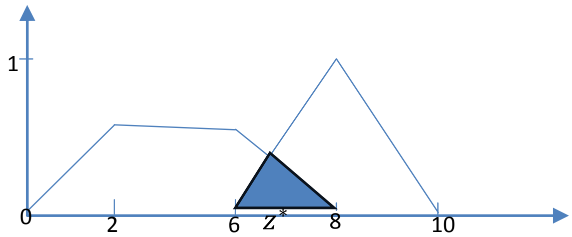
$$z^* = \frac{\sum_{k=1}^{n} \mu_{\widetilde{C_k}}(z) \square_z \bar{z} dz}{\sum_{k=1}^{n} \mu_{\widetilde{C_k}}(z) \square_z dz},$$



First membership function



Second membership function



Defuzzification

(vi) Center of Largest Area:

The center of largest area method is applied when the fuzzy output set has two or more than two non-overlapping convex subregions. Thede-fuzzified value z^* is calculated by using the formula

$$z^* = \frac{\int \mu_{\widetilde{C_m}}(z) \cdot z \, dz}{\int \mu_{\widetilde{C_m}}(z) \, dz},$$

where $\widetilde{C_m}$ is the convex subregion that has the largest area making up $\widetilde{C_k}$. This condition is applicable in the case when the overall output $\widetilde{C_k}$ is non-convex. And in the case, when $\widetilde{C_k}$ is convex, z^* is the same quantity as determined by the centroid method or the center of largest area method.

(vii) First of Maxima (or Last of Maxima):

In this method,the largest height in the union of fuzzy output sets $\widetilde{C_k}$ is defined as;

$$hgt(\widetilde{C_k}) = \sup_{z \in Z} \mu_{\widetilde{C_k}}(z).$$

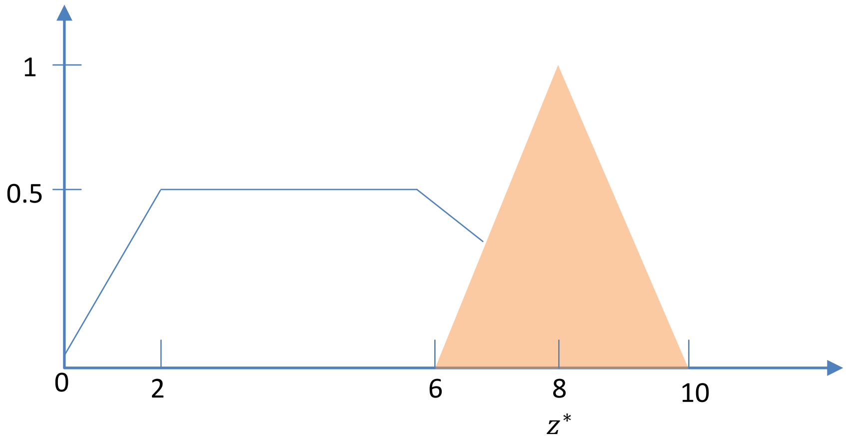
In the first of maxima method, the de-fuzzified value z^*, is given as;

$$z^* = \inf_{z \in Z}\{z \in Z: \mu_{\widetilde{C_k}}(z) = hgt(\widetilde{C_k})\}.$$

And, in the last of maxima method,the de-fuzzified value z^*, is given by the expression

$$z^* = \sup_{z \in Z}\{z \in Z: \mu_{\widetilde{C_k}}(z) = hgt(\widetilde{C_k})\}.$$

Here, inf (infimum) is the greatest lower bound (glb) and sup (supremum) is the least upper bound (lub).



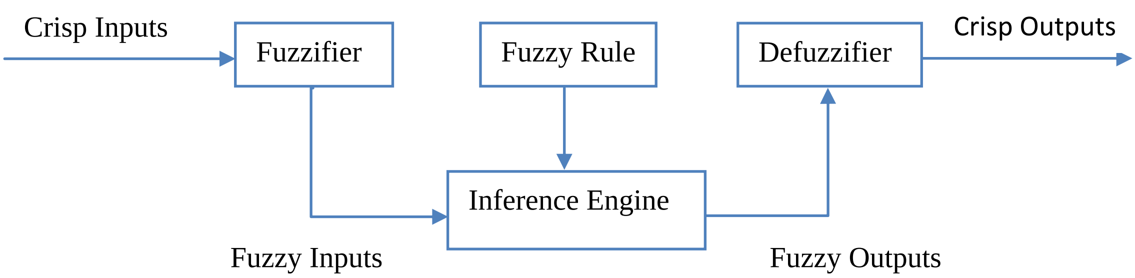
9. Fuzzy Rule:

Human beings develop some rules in their mind to control any activities on the basis of their own intelligence and these rules can be converted into fuzzy rules. So, the rule made by traffic police to manage traffic jam can be converted into fuzzy rules. Using these rules, Traffic Light Controller works properly.

10. Traffic Light Controller:

A Traffic Light Controller is a Fuzzy Logic Controller. So, we introduce Fuzzy Logic Controller to develop a Traffic Light Controller. Now, let us see, **"How the Fuzzy Logic Controller works?"**

At first, we insert crisp inputs to fuzzifier and this fuzzifier converts these crisp inputs to fuzzy inputs with the process of fuzzification. Now the obtained fuzzy inputs are given to the Inference Engine where this engine converts fuzzy inputs into fuzzy outputs according to the given fuzzy rules. At last, these fuzzy outputs are given to the Defuzzifier to get crisp outputs. These crisp outputs are the required outputs.



11. Conclusion:

This paper includes the process of Fuzzification and Defuzzification. Since fuzzification and defuzzification are the important parts of Fuzzy Logic, therefore a Traffic Light Controller may be developed by developing Fuzzy Logic Controller with the process of fuzzification and defuzzification using fuzzy rules. The main aim of this paper is to describe the fuzzification of crisp inputs into fuzzy inputs and the defuzzification of obtained outputs of TLC into crisp outputs. Here methods of defuzzification are explained one by one. Although, centroid method is often used to defuzzify the fuzzy outputs.

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