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Solving Delay Differential Equations Using Mohand Transform

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Abstract
An integral transform named as Mohand Transform which is used for solving non-linear Delay differential equations (DDEs). Here, outcome is attained as a succession by assigning the Mohand transform to the nonlinear delay differential equations and later disintegrate nonlinear term to find out Adomnian polynomial. The results obtained by this method are quite effective and reliable.

Keyword Non-linear delay differential equation, Mohand Transform, Approximate solution.

1. Introduction

Delay differential equation has an important contribution in Physics,Chemistry, Applied mathematics, Population dynamics,Physiology applications, Engineering and especially in the field of bioscience.Mohand transformation is used to solve DDEs of the type:

$$x'(t) = f(t, x, x(t-\tau)); \;\; t > t_o \qquad (1)$$

$$x(t) = \phi(t) \quad ; \; t \le t_o$$

Here, $\phi(t)$ denotes initial function, $\tau(t, x(t))$ is termed as delay. It is both time dependent and state dependent delay. As it depends on time it is time dependent and when depends on both state and time it is termed as state dependent andif independent then it is constant delay.

Different types of methods exist to find out an exact and approximate solution such as variational iterative method [10], Adomian decomposition method (ADM) in 2007 [9], Decomposition method [8], Runga-kutta method [11], Variable multistep method and also there areso many integral transforms tosolve boundary value problems. Transform was firstly introduced by T.S. Stieltjes. Oldest method is Laplace transform method with sumudu and most used and convenient method. Laplace transform is the oldest method with Sumudu to solve initial value integral transform and the most used. But here we use a new integral transform method which is known as Mohand transform method which is a basis for a possible number of integral transforms. Initial approximation and recursion formula are used to compute the components. Mohand transformation method does not perceive confined transformations, perturbation, linearization or discretization. This paper is classified as follows. Part 2 is based on Mohand transform and its properties. Part 3 is based onMohand transform method which is used for find the components for nth order DDEs. Part 4 provides vision to numerical applications of nonlinear DDEs.Part 5 is provides conclusion.

2. Mohand Transform and Elementary Properties:-

To simplify the means of differential equations like ordinary and partial in the time estatea transform is introduced by Mohand Mahgoub which is known as Mohan transform. It is originated from the classical Fourier integral.So, Mohand transform is defined for function of exponential order; we assume functions in the set A are specified as:

$$A=\left\{ f(t):\exists M,k_1,k_2>0,\left|f(t)\right|<Me^{\frac{|t|}{k_j}};if\ t\varepsilon(-1)^j\times\left[0,\infty\right) \right\}$$

Here, M is a constant and finite number, k_1, k_2 , either finite or infinite. Mohand transform denotes the operator M(·)which is given below.

$$M\left[f(t)\right] = R\left(v\right)=v^2\int_0^\infty f(t)e^{-vt}dt;t\geq 0,\ k_1\leq v\leq k_2 \qquad (2)$$

Here in this transform variable v is accustomed to represent the variable t in the contention of the function f. It is interrelated to Fourier, Laplace and Elzaki transforms.

Such functions of Mohand transforms are specified as:

i. $M\left[1\right]=v$

ii. $M\left[t\right]=1$

iii. $M\left[t^n\right]=\dfrac{n!}{v^{n-1}}$

iv. $M\left[e^{at}\right]=\dfrac{v^2}{v-a}$

v. $M\left[\sin at\right]=\dfrac{av^2}{a^2+v^2}$

vi. $M\left[\cos at\right]=\dfrac{v^3}{a^2+v^2}$

Now the derivatives for Mohand transform are:

$$M\left[f'(t)\right]=vR\left(v\right)-v^2 f(0)$$
$$M\left[f''(t)\right]=v^2R\left(v\right)-v^3 f(0)-v^2 f'(0)$$
$$M\left[f^n(t)\right]=v^nR\left(v\right)-\sum_{k=0}^{n-1}v^{n-k+1}f^k\left(0\right)$$

3.Mohand Transform method

Mohand Decomposition methodis the amalgamation of Mohand Transform [7] and Adomnian Decomposition method [9] is used for finding out solutions of nonlinear DDEs. As we apply Mohand transform to DDEs, we get a solution in series form and we can easily find Adomnian polynomials by decomposing the nonlinear term.

Consider the nonlinear DDE of the form:

$$x'=f(t,x,x(t-\tau)) \; ; \qquad x(0)=\alpha \qquad (3)$$

Applying Mohand transform and using initial condition on (3) we get,

$$M\left[x'\right]=M\left[f(t,x,x(t-\tau))\right]$$
$$M\left[x(t)\right]=\alpha v+\frac{1}{v}M\left[f(t,x,x(t-\tau))\right]$$

So, nonlinear term $N\left(x,x_\tau\right)$ can be break down into an infinite series of polynomials .These polynomials are known as Adomnian polynomials which is symbolized as $\sum_{n=0}^{\infty}A_n$ where A_n is conventionally approved to the computed form

$$A_n=\frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{i=0}^{\infty}\lambda^i x_i;\sum_{i=0}^{\infty}\lambda^i x_{\tau i}\right)\right]_{\lambda>0}$$

Now this approaching method equips the solution in form of an infinite series,

$$M\left[\sum_{n=0}^{\infty}x_n\right]=\alpha v+\frac{1}{v}M\left[\sum_{n=0}^{\infty}A_n\right] \qquad (4)$$

Using (4), we have recursive algorithm which is follow as:

$$M\left[x_0\right]=\alpha v$$
$$M\left[x_{n+1}\right]=\frac{1}{v}M\left[A_n\right],\; n>0$$

Using inverse transform, we get $x_0,x_1,x_2,......$

The systematic solution of nonlinear DDEs is in terms of infinite series

$$x(t)=\sum_{n=0}^{\infty} x_n(t)$$

By finding adequate no's of x_n we get better result with better certainty.

Numerical Illustration:-

Problem 4.1:

Let us take DDE of first order

$$x'(t)=1-2x^2\left(\frac{t}{2}\right),\, x(0)=0$$

Exact solution of given equation is $x(t)=\sin t$.

Applying Mohand Transform to the given equation, we get

$$M\left[x(t)\right] = \quad 1-\frac{2}{v}M\left[x^2\left(\frac{t}{2}\right)\right]$$

By using Adomian Decomposition Method, we get

$$M\left[\sum_{n=0}^{\infty} x_n(t)\right]=1-\frac{2}{v}M\left[\sum_{n=0}^{\infty} A_n\right] \qquad (5)$$

Using eqn. (5), we get

$$x_0(t)=t$$

$$x_1(t)=-\frac{t^3}{3!}$$

$$x_2(t)=\frac{t^5}{5!}$$

$$x_3(t)=-\frac{t^7}{7!}$$

The infinite series solution becomes,

$$x=x_0+x_1+x_2+x_3+...$$

$$x=t-\frac{t^3}{3!}+\frac{t^5}{5!}-\frac{t^7}{7!}+....$$

That tends to exact solution $x(t)=\sin t$ as $n\rightarrow\infty$.

Problem 4.2:

Let us take third order nonlinear DDE

$$x'''(t)=-1+2x^2\left(\frac{t}{2}\right),\, x(0)=0,\, x'(0)=1,\, x''(0)=0$$

Exact solution of given equation is $x(t)=\sin t$.

Applying Mohand Transform to the given equation, we get

$$M\left[x(t)\right] = \quad 1-\frac{1}{v^2} + \frac{2}{v^3}M\left[x^2\left(\frac{t}{2}\right)\right]$$

By using Adomian Decomposition Method, we get

$$M\left[\sum_{n=0}^{\infty}x_n(t)\right]=1-\frac{1}{v^2} + \frac{2}{v^3}M\left[\sum_{n=0}^{\infty}A_n\right] \qquad (6)$$

From Eqn. (6), we get

$$x_0(t) = t - \frac{t^3}{3!}$$

$$x_1(t) = \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{5}{8}\frac{t^9}{9!}$$

$$x_2(t) = \frac{3}{8}\frac{t^9}{9!} - \frac{63}{64}\frac{t^{11}}{11!} + \frac{505}{1024}\frac{t^{13}}{13!} - \frac{275}{2048}\frac{t^{15}}{15!}$$

The infinite series solution becomes,

$$x = x_0 + x_1 + x_2 + ...$$

$$x = t - \frac{t^3}{3!} + \frac{t^5}{5!} - \frac{t^7}{7!} + \frac{t^9}{9!} - ....$$

That tends to exact solution $x(t) = \sin t$ as $n \to \infty$.

Conclusions

Here, we introduced Mohand transform with adomnian decomposition to solve nonlinear delay differential equations. Mohand decomposition method provides estimated solutions to the nonlinear DDEs. This method provides quickly convergent consecutive approximations by recursive relations. This is efficient method to solve nonlinear DDEs with better efficacy.

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