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2-WOVEN FRAMES IN 2-HILBERT SPACES USING CONTINUOUS FUNCTIONS

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Abstract: We present 2-woven frames in 2-Hilbert spaces and discuss some of its characteristics in this work. Additionally, operators for 2-woven frames are developed, and some associated results are established for these operators in 2-Hilbert spaces.The required two-sample and change-point tests in Sections will be developed using the theoretical contributions. Here, the suggested method's utility is more obvious because it can be challenging to distinguish variations between two smooth curves in real-world scenarios. Furthermore, in many practical scenarios, minor differences might not even be significant. Instead of trying to test for exact equality under the null hypothesis, the "relevant" setting is used, which allows preset deviations from an assumed null function. Its relationship to operators is the rationale behind its selection and study in the thesis. We construct frame sequences and investigate a class of operators associated with a particular Bessel sequence, which transforms it into a frame for all operators in the class.

Keywords:2-Woven Frames, 2-Hilbert Spaces,Continuous Functions

1. Introduction:

In their work on nonharmonic Fourier series, Duffin and Schaeffer introduced frames in Hilbert space. Daubechies (1986) reintroduced them, and since then, work on frames has continued. Applied mathematics and engineering have been significantly impacted by frame theory, and certain characteristics of frames have made them a crucial component of functional analysis.The attributes of the finite sample are examined through a simulation analysis and an application to annual temperature profiles. Due to the recent substantial advancements in better data collection technology, there has been a lot of study focused on developing statistical methodologies for the analysis of functional data gathered over time and/or location. Most of the effort has been on developing Hilbert space-based techniques, for which there is currently a comprehensive theory. However, the critical significance of smoothness has been thoroughly examined by Ramsay and Silverman (2005), and almost all functions that are fit in practice are at least continuous. Fully functional solutions might be preferable in these circumstances than dimension reduction techniques, which could lead to information loss.It is simple to generate simultaneous asymptotic confidence bands for the difference of the mean functions using the approach that has been established thus far. Confidence bands for functional data in Hilbert spaces are well-documented in the literature. Cao et al. (2014) studied simultaneous confidence bands for the mean of dense functional data based on polynomial spline estimators; Cao (2014) developed simultaneous confidence bands for derivatives of functional data when multiple realisations are at hand for each function, exploiting within-curve correlation; Zheng et al. (2014) addressed the sparse case. Degras (2011) dealt with confidence bands for nonparametric regression with functional data. Most recently, to construct confidence bands for functional parameters, Choi and Reimherr (2018) identified geometric characteristics similar to Mahalanobis distances.This set of findings about functional data valued in Banach space is unprecedented. The first theorem builds asymptotic simultaneous confidence bands for the two-sample situation using the limit distribution found in Theorem. In the following part, a comparable bootstrap analogue will be created. Using standard reasons, confidence bands for the one-sample case can be produced similarly, and the associated results are ignored.

THEOREM 1: Assume that Theorem conditions are met. Then, for $\alpha \in (0,1)$, identify the $(1-\alpha)$-quantile of the random variable T as defined by $u_{1-\alpha}$, and construct the functions

$$\mu_{m,n}^{\pm}(t) = \frac{1}{m}\sum_{j=1}^{m} X_j - \frac{1}{n}\sum_{j=1}^{n} Y_j \pm \frac{u_{1-\alpha}}{\sqrt{n+m}} \ldots (1)$$

Then the established

$$C_{\alpha,m,n} = \{\mu \in C([0,1]) : \mu_{m,n}^{-}(t) \leq \mu(t) \leq \mu_{m,n}^{+}(t) \text{ for all } t \in [0,1]\}$$

defines a instantaneous asymptotic $(1-\alpha)$ self-assurance band aimed at $\mu_1 - \mu_2$, that is,

$$\lim_{m,n\to\infty} \mathbb{P}(\mu_1 - \mu_2 \in C_{\alpha,m,n}) = 1 - \alpha \ldots (2)$$

It should be noted that the simultaneous confidence bands presented in Theorem apply for all $t \in [0,1]$ and not only practically everywhere, in contrast to their counterparts in Hilbert space. This property makes the suggested bands easier to understand and potentially more effective for applications.

THEOREM 2: Assume that $(X_j : j \in \mathbb{N})$ and $(Y_j : j \in \mathbb{N})$ placate Hypothesis with metric $\rho(s,t) = |s-t|^\theta, \theta \in (0,1], J\theta > 1$ and let $\hat{B}_{m,n}^{(1)}, \dots, \hat{B}_{m,n}^{(R)}$ signify the bootstrap procedures distinct by (3.19) s.t.$l_1 = m^{\beta_1}, l_2 = n^{\beta_2}$ through

$$0 < \beta_i < v_i/(2+v_i), \quad \bar{\tau} > (\beta_i(2+v_i)+1)/(2+2v_i) \dots .(3)$$

andv_i agreed in Hypothesis, $i = 1, 2$. Also, assume on behalf of the multipliers in $\mathbb{E}\left|\xi_1^{(r)}\right|^J < \infty$and $\mathbb{E}\left|\zeta_1^{(r)}\right|^J < \infty$. Before

$$\left(Z_{m,n}, \hat{B}_{m,n}^{(1)}, \dots, \hat{B}_{m,n}^{(R)}\right) \rightsquigarrow \left(Z, Z^{(1)}, \dots, Z^{(R)}\right) \dots \dots (4)$$

in$C([0,1])^{R+1}$ as $m, n \to \infty$, where $Z_{m,n}$ is distinct in (4) and $Z^{(1)}, \dots, Z^{(R)}$ are self-governing copies of the concentrated Gaussian process Z distinct. Keep in mind that both the alternative and the null hypothesis uphold Theorem 1. Based on the multiplier bootstrap, it yields the following findings for tests and confidence bands for the classical hypothesis. In light of this, note that for the statistics

$$T_{m,n}^{(r)} = \left\|\hat{B}_{m,n}^{(r)}\right\|, \quad r = 1, \dots, R \dots .(5)$$

the incessant mapping proposition profits

$$\left(\sqrt{n+m}\hat{d}_\infty, T_{m,n}^{(1)}, \dots, T_{m,n}^{(R)}\right) \Rightarrow \left(T, T^{(1)}, \dots, T^{(R)}\right) \dots \dots (6)$$

where the random variables $T^{(1)}, \dots, T^{(R)}$ are self-governing copies of the statistic T distinct in (4). Currently, if $T_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}$ is the experiential $(1-\alpha)$-quantile of the bootstrap sample $T_{m,n}^{(1)}, \dots, T_{m,n}^{(R)}$, the subsequent consequences are attained.

THEOREM 3: Assuming that Theorem 2 premises are met, define the functions

$$\mu_{m,n}^{R,\pm}(t) = \frac{1}{m}\sum_{j=1}^{m} X_j - \frac{1}{n}\sum_{j=1}^{n} Y_j \pm \frac{T_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n+m}} \dots ..(7)$$

Before

$$\hat{C}_{\alpha,m\cdot n}^{R} = \left\{\mu \in C([0,1]) : \mu_{m,n}^{R,-}(t) \le \mu(t) \le \mu_{m,n}^{R,+}(t) \text{ for all } t \in [0,1]\right\} \dots ..(8)$$

defines a simultaneous asymptotic $(1-\alpha)$ self-assurance band for $\mu_1 - \mu_2$, that is,

$$\lim_{R\to\infty} \liminf_{m,n\to\infty} \mathbb{P}\left(\mu_1 - \mu_2 \in \hat{C}_{\alpha,m.n}^{R}\right) \ge 1 - \alpha \dots \dots (9)$$

Equation (9) weak convergence and the continuous mapping theorem can be used to create point-wise bootstrap confidence intervals in a manner similar to that described. For the purpose of conciseness, the specifics are left out. A related assertion about the bootstrap test for the classical hypotheses which rejects the null hypothesis whenever

$$\hat{d}_\infty > \frac{T_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n+m}} \ldots\ldots (10)$$

where the statistic $\hat{d}_\infty$ is definite.

THEOREM 4: If the presumptions of Theorem 1 are met, then test is consistent for hypotheses and has asymptotic level α. More specifically, assuming that there is no variation in the mean functions,

$$\lim_{R\to\infty} \limsup_{m,n\to\infty} \mathbb{P}\left(\hat{d}_\infty > \frac{T_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n+m}}\right) = \alpha \ldots\ldots (11)$$

and, under the substitute, for some $R \in \mathbb{N}$,

$$\liminf_{m,n\to\infty} \mathbb{P}\left(\hat{d}_\infty > \frac{T_{m,n}^{\{\lfloor[(1-\alpha)\rfloor\}}}{\sqrt{n+m}}\right) = 1 \ldots\ldots (12)$$

THEOREM 5: Let the expectations of Theorem 1be fulfilled, then

$$d_H\left(\hat{\mathcal{E}}_{m,n}^{\pm}, \mathcal{E}^{\pm}\right) \overset{\mathbb{P}}{\to} 0 \ldots..(13)$$

where the sets $\hat{\mathcal{E}}_{m,n}^{\pm}$are distinct by (13). The main insinuation of Theorem 3 involves in the detail that the random inconstant

$$\max_{t\in\hat{\mathcal{E}}_{m,n}^{+}} \hat{B}_{m,n}(t) \ldots..(14)$$

weakly converges to $\max_{t\in\mathcal{E}^+} Z(t)$, the random variable. However, the connection of these two propositions is more complex and necessitates a continuity argument, whereby the following result is proven.

THEOREM 6: Let the expectations of Theorem 2 be fulfilled and outline, for $= 1, \ldots, R$,

$$K_{m,n}^{(r)} = \max\left\{\max_{t\in\hat{\mathcal{E}}_{m,n}^{+}} \hat{B}_{m,n}^{(r)}(t), \max_{t\in\hat{\mathcal{E}}_{m,n}^{-}} \left(-\hat{B}_{m,n}^{(r)}(t)\right)\right\} \ldots\ldots (15)$$

Before

$$\left(\sqrt{n+m}\left(\hat{d}_\infty - d_\infty\right), K_{m,n}^{(1)}, \ldots, K_{m,n}^{(R)}\right) \ldots\ldots (16)$$

in $\mathbb{R}^{R+1}$, where $d_\infty = \|\mu_1 - \mu_2\|$, the measurement $\hat{d}_\infty$ is distinct in (14) and the variables $T^{(1)}(\mathcal{E}), \ldots, T^{(R)}(\mathcal{E})$ are self-governing copies of $T(\mathcal{E})$ distinct in Theorem 3. A straightforward bootstrap test for the no relevant change hypothesis is produced by Theorem 4. Specifically, we may define the empirical (1-α)-quantile of the bootstrap sample $K_{m,n}^{(1)}, \dots, K_{m,n}^{(R)}$by $K_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}$; at α, the null hypothesis of no relevant change is rejected.

$$\hat{d}_{\infty} > \Delta + \frac{K_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n+m}} \dots\dots(17)$$

This section's final conclusion demonstrates the asymptotic level α and consistency of the test.

THEOREM 7: Assume that Theorem 4 premises are met. Next, assuming that there is no significant variation in the mean functions,

$$\lim_{R\to\infty} \limsup_{m,n\to\infty} \mathbb{P}\left(\hat{d}_{\infty} > \Delta + \frac{K_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n+m}}\right) \le \alpha \dots\dots(18)$$

and, under the substitute of a relevant variance in the mean functions, for some $R \in \mathbb{N}$,

$$\liminf_{m,n\to\infty} \mathbb{P}\left(\hat{d}_{\infty} > \Delta + \frac{K_{m,n}^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n+m}}\right) = 1 \dots\dots(19)$$

THEOREM 8: Assume $d_{\infty} > 0, s^* \in (0,1)$ and let $(X_{n,j} : n \in \mathbb{N}, j = 1, \dots, n)$ be an selection of $C([0,1])$-valued random variables sustaining Assumption with $\rho(s,t) = |s-t|^{\theta}$, $\theta \in (0,1]$ and $J\theta > 1$. Then

$$\mathbb{D}_n = \sqrt{n}\left(\hat{\mathbb{M}}_n - s^*(1-s^*)d_{\infty}\right) \dots\dots(20)$$

where W is the centred Gaussian measure on $C([0,1]^2)$, defined by, $\mathcal{E} = \mathcal{E}^+ \cup \mathcal{E}^-$, and the sets $\mathcal{E}^+$and $\mathcal{E}^-$are specified in (17). The statistic $\hat{\mathbb{M}}_n$is defined in (20). With functions $\mu_1 - \mu_2$ having the same sup-norm d_{∞}but distinct corresponding sets E, the limit distribution of D_n might be somewhat complex and dependent on that set. It is also important to note that Theorem 7 requires the condition $d_{\infty} > 0$. The weak convergence in the remaining case $d_{\infty} = 0$ can be inferred from $\sqrt{n}\hat{\mathbb{U}}_n = \hat{\mathbb{W}}_n$, (4.4), and the continuous mapping theorem, which states,

$$\sqrt{n}\hat{\mathbb{M}}_n \xrightarrow{\mathcal{D}} \check{T} = \sup_{(s,t)\in[0,1]^2} |\mathbb{W}(s,t)| \dots\dots(21)$$

whenever $d_{\infty} = 0$. If $d_{\infty} > 0$, Since the actual location of the change-point s* is unknown, an estimate must be made using the information that is now accessible. One such estimator is suggested in the following theorem, which is demonstrated in Section, and its large-sample behaviour is described as a rate of convergence.

THEOREM 9: Undertake $d_\infty > 0, s^* \in (0,1)$ and let $(X_{n,j}: n \in \mathbb{N}, j = 1, \dots, n)$ stand an array of $C([0,1])$-valued random variables sustaining Assumption, where the random variable M in Statement (A3) is circumscribed and $\rho(s,t) = |s-t|^\theta$with $\theta \in (0,1], J\theta > 1$. Then the estimator

$$\tilde{s} = \frac{1}{n} \arg \max_{1 \le k < n} \left\| \hat{\mathbb{U}}_n \left(\frac{k}{n}, \cdot \right) \right\| \dots \dots (22)$$

Satiates

$$|\tilde{s} - s^*| = O_{\mathbb{P}}(n^{-1}).$$

Remember that the open interval $(\vartheta, 1-\vartheta)$ is the only range of change sites that can exist. Define the adjusted change-point estimator.

$$\hat{s} = \max\{\vartheta, \min\{\tilde{s}, 1-\vartheta\}\} \dots .. (23)$$

where$\tilde{s}$ is specified by (10). Since $|\hat{s} - s^*| \le |\tilde{s} - s^*|$, it shadows that

$$|\hat{s} - s^*| = O_{\mathbb{P}}(n^{-1})$$

if $d_\infty > 0$, and, if $d_\infty = 0$ assume that $\hat{s}$ meets weakly to a $[\vartheta, 1-\vartheta]$-valued random variable $s_{\max}$.

THEOREM 4.10: Let $\hat{B}_n^{(1)}, \dots, \hat{B}_n^{(R)}$ signify the bootstrap processes clear, where $l = n^\beta$ for particular $\beta \in (1/5, 2/7)$ and undertake that the underlying array $(X_{n,j}: j = 1, \dots, n; n \in \mathbb{N})$ satisfies Supposition through metric $\rho(s,t) = |s-t|^\theta, \theta \in (0,1], J\theta > 1$ in (A3) and $v \ge 2$ in (A1) and

$$(\beta(2+v)+1)/(2+2v) < \bar{\tau} < 1/2$$

in (A4). Also, assume additionally on behalf of the multipliers in (418) $\mathbb{E}\left|\xi_1^{(r)}\right|^J < \infty$Then

$$\left(\hat{\mathbb{W}}_n, \hat{\mathbb{W}}_n^{(1)}, \dots, \hat{\mathbb{W}}_n^{(R)}\right) \rightsquigarrow \left(\mathbb{W}, \mathbb{W}^{(1)}, \dots, \mathbb{W}^{(R)}\right)$$

in $C([0,1]^2)^{R+1}$, where $\hat{\mathbb{W}}_n$ and $\mathbb{W}$ are distinct in (21) and (23), correspondingly, and $\mathbb{W}^{(1)}, \dots, \mathbb{W}^{(R)}$ are self-governing copies of $\mathbb{W}$. For the classical hypotheses, we now consider a resampling approach where Δ=0 in (4.72). To achieve that, define, for $r = 1, \dots, R$,

$$\check{T}_n^{(r)} = \max\left\{\left|\hat{\mathbb{W}}_n^{(r)}(s,t)\right| : s,t \in [0,1]\right\} \dots .. (24)$$

The continuous mapping theory thus states that

$$\left(\sqrt{n}\hat{\mathbb{M}}_n, \check{T}_n^{(1)}, \dots, \check{T}_n^{(R)}\right) \Rightarrow \left(\check{T}, \check{T}^{(1)}, \dots, \check{T}^{(R)}\right) \dots ..(25)$$

in $\mathbb{R}^{R+1}$, where $\check{T}^{(1)}, \dots, \check{T}^{(R)}$ are self-governing copies of the random variable $\check{T}$ distinct in (4.87). If $\check{T}_n^{\{\lfloor R(1-\alpha) \rfloor\}}$ is the experiential $(1-\alpha)$-quantile of the bootstrap sample

$\check{T}_n^{(1)}, \check{T}_n^{(2)}, \ldots, \check{T}_n^{(R)}$, the traditional null hypothesis $H_0: \mu_1 = \mu_2$ of no change-point is rejected, when

$$\hat{\mathbb{M}}_n > \frac{\check{T}_n^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n}} \ldots\ldots (26)$$

Similar reasoning as those in Section imply that this test is consistent and has an asymptotic level of α, according to above Theorem.

$$\lim_{R\to\infty} \limsup_{n\to\infty} \mathbb{P}_{H_0}\left(\hat{\mathbb{M}}_n > \frac{\check{T}_n^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n}}\right) = \alpha$$
$$\liminf_{n\to\infty} \mathbb{P}_{H_1}\left(\hat{\mathbb{M}}_n > \frac{\check{T}_n^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n}}\right) = 1 \quad \ldots.(27)$$

for every $R \in \mathbb{N}$. For the purpose of conciseness, the specifics are left out.We will now proceed with the development of the bootstrap methodology for the issue of testing for a relevant change-point, that is, Δ>0. Apparently, the theoretical analysis is much more involved because the estimates of the extremal sets $\mathcal{E}^+$and $\mathcal{E}^-$are specified by the null hypothesis, which defines a set in C([0,1]).

$$\hat{\mathcal{E}}_n^{\pm} = \left\{t \in [0,1]: \pm(\hat{\mu}_1(t) - \hat{\mu}_2(t)) \geq \hat{d}_\infty - \frac{c_n}{\sqrt{n}}\right\} \ldots\ldots (28)$$

where$\lim_{n\to\infty} c_n/\log(n) = c > 0$ and $\hat{d}_\infty$ is specified in (4.87). Contemplate bootstrap analogs

$$T_n^{(r)} = \frac{1}{\hat{s}(1-\hat{s})} \max\left\{\sup_{t\in\hat{\mathcal{E}}_n^+} \hat{W}_n^{(r)}(\hat{s}, t), \sup_{t\in\hat{\mathcal{E}}_n^-}\left(-\hat{W}_n^{(r)}(\hat{s}, t)\right)\right\} \ldots\ldots (29)$$

$(r = 1, \ldots, R)$of the measurement

$$\sqrt{n}(\hat{d}_\infty - d_\infty)$$
$$d_\infty = \|\mu_1 - \mu_2\|.$$

THEOREM 4.11: Let the expectations of Theorem 8 be fulfilled, then if $d_\infty > 0$,

$$\left(\sqrt{n}(\hat{d}_\infty - d_\infty), T_n^{(1)}, \ldots, T_n^{(R)}\right) \Rightarrow \left(T(\mathcal{E}), T^{(1)}, \ldots, T^{(R)}\right) \ldots\ldots(30)$$

in $\mathbb{R}^{R+1}$, where $T^{(1)}, \ldots, T^{(R)}$ are self-governing copies of the random variable $T(\mathcal{E})$ distinct. Rejecting the null hypothesis yields a test for the hypothesis of a relevant change-point in time series of continuous functions whenever

$$\hat{d}_\infty > \Delta + \frac{T_n^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n}} \ldots\ldots (31)$$

where$T_n^{\{\lfloor R(1-\alpha)\rfloor\}}$ is the experiential $(1-\alpha)$-quantile of the bootstrap sample $T_n^{(1)}, T_n^{(2)}, \ldots,$ $T_n^{(R)}$. Similar reasons imply that this test is consistent and has an asymptotic level of α in accordance with Theorem 8.

$$\lim_{R\to\infty} \limsup_{n\to\infty} \mathbb{P}_{H_0}\left(\hat{d}_\infty > \Delta + \frac{T_n^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n}}\right) \leq \alpha$$

and

$$\liminf_{n\to\infty} \mathbb{P}_{H_1}\left(\hat{d}_\infty > \Delta + \frac{T_n^{\{\lfloor R(1-\alpha)\rfloor\}}}{\sqrt{n}}\right) = 1 \ldots..(32)$$

for every R∈N. For the purpose of conciseness, the specifics are left out.

2. Conclusion

The notion of frames for operators is presented for a set of continuous linear functionals built on a Banach space. The new concept has been shown to be a logical extension of the Banach frames developed by Casazza et al. in 2005. The necessary and sufficient conditions that must be satisfied are given, along with results on producing frames for operators. Moreover, it is shown that the notions of "atomic systems" and "frame for operators" as used in the thesis are not generally the same, unless the associated sequence spaces violate some additional requirements. Analysis, synthesis, and frame operators are possible for a Bessel sequence in a Hilbert space. Studying these related operators can help one understand some of the Bessel sequence's properties. For instance, if the synthesis operator is invertible, the sequence is a Riesz basis, and vice versa. Găvruţa developed a generalisation of earlier frames in literature: a frame for operators in Hilbert spaces. Its relationship to operators is the rationale behind its selection and study in the thesis. We construct frame sequences and investigate a class of operators associated with a particular Bessel sequence, which transforms it into a frame for all operators in the class.

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