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Electric And Scalar Charged Fluid Sphere in General Relativity

Dr. Sujit Kumar
Research Scholar
University Department of Mathematics
M.U. Bodh-Gaya (Bihar)

ABSTRACT:

In the present paper we have investigated interior solution for an electric and scalar charged fluid sphere using some suitable assumptions in general theory of relativity. We have also found and discussed pressure, mass density, charge density and scalar charge density along with their values at the centre.

Key Words : Charged fluid, scalar field, mass density, pressure, electric potential.

1. Introduction

Som and Bedran [7] have discussed a class of solutions for static incoherent spherical dust distribution with spin. Charged fluid spheres have been studied many researchers [1, 3, 4, 6, 10]. An interior solution of electroscalarly charged static 'dust' sphere has been found by Teixeira et. al. [9] and it has been shown that the geometric mass of an electroscalarly charged sphere is

$$m = (q^2 - \gamma_b^2)^{1/2}$$

where q is the electric charge, b the scalar charge and $\gamma = \pm 1$.

Florides [2] has shown that the geometric mass of a charged sphere is

$$m = \mu(a) + \varepsilon)(a),$$

where, μ (a) is the contribution from the mass density and ε (a) is that from electric charge. Som et. al. [8] have found Reissner Nordstrom solution from the Schwarzschild solution by a co-ordinate transformation and have shown that the geometric mass of a charged sphere is

$$m = \left(m_s^2 + q^2\right)^{1/2},$$

where, m_S is the Schwarzschild mass and q is the charge of the sphere. Paul [5] has obtained an interior solution of electroscalarly charged fluid sphere and has shown that the geometric mass of the sphere has contribution from its mass density and scalar electric charges. In the present paper, we have investigated the interior solution of an electric and scalar charged fluid sphere in general relativity under certain assumptions.

2. The Field equations and Their Solutions :

We consider the spherically symmetric metric given by

(2.1) $$ds^2 = e^{2\eta}dt^2 - e^{2\alpha}dr^2 - r^2e^{\alpha-\eta}d\theta^2 - r^2e^{\alpha-\eta}\sin^2\theta d\phi^2$$

where r, θ, ϕ and t are numbered 1, 2, 3 and 4 respectively. Here α and η are functions of alone.

Einstein – Maxwell scalar field equations are

(2.2) $$R_\mu^v = -8\pi\left(T_\mu^v - \frac{1}{2}\delta_\mu^v T\right)$$

(2.3) $$T_\mu^v = (\rho + p)u_\mu u^v - \delta_\mu^v P + \frac{1}{4\pi}\left[-F^{v\alpha}F_{\mu\alpha} + \frac{1}{4}\delta_\mu^v F^{\alpha\beta}F_{\alpha,\beta}\right] + k_\mu^v$$

(2.4) $$4\pi\gamma K_\mu^v = s^{;v}s_{;\mu} - \frac{1}{2}\delta_\mu^v S^{;\alpha}S_{;\alpha},$$

(2.5) $$F^{\mu v};\nu = 4\pi\,\sigma u^\mu,$$

(2.6) $$F_{[\mu\nu:\alpha]} = 0$$

(2.7) $$S_{;\mu}^{;\mu} = -4\pi\gamma\beta$$

where ρ, P, σ and β are the mass density, pressure, charge density and scalar charge density respectively s is the scalar field. The matter is at rest in the co-ordinate system (2.1) so that

$$u^{\mu} = \delta_4^{\mu}\left(\delta_{44}\right)^{-1/2}$$

The electric field is such that only $F_{41} = +\psi_1$ exists along radial direction where ψ is the electric potential. The suffix 1 indicates differentiation with respect to r equation (2.2) with the help of equation (2.1) gives

(2.8) $$\eta_{11} + \frac{2\eta_1}{r} = 4\pi\left(\rho + 3P\right)e^{2\alpha} + e^{-2\eta}\psi_1^2,$$

(2.9) $$\alpha_{11} - \frac{\alpha_1^2}{2} - \alpha_1\eta_1 + \frac{3}{2}\eta_1^2 - \frac{2\eta_1}{r}$$

$$= -4\pi\left(\rho - P\right)e^{2\alpha} + e^{-2\eta}\psi_1^2 - 2\gamma S_1^2,$$

(2.10) $$\frac{1}{2}\eta_{11} - \frac{1}{2}\alpha_{11} + \frac{\eta_1}{r} - \frac{\alpha_1}{\alpha} - \frac{1}{r^2} + \frac{1}{r^2}e^{\eta+\alpha}$$

$$= 4\pi\left(\rho - P\right)e^{2\alpha} + e^{-2\eta}\psi_1^2,$$

(2.11) $$\frac{d}{dr}\left(r^2 e^{-2\eta}\psi_1\right) = -4\pi\sigma r^2 e^{2\alpha-\eta},$$

(2.12) $$\frac{d}{dr}\left(r^2 S_1\right) = -4\pi\gamma\beta r^2 e^{2\alpha}$$

Now since there are five equations and eight variables, let us assume

(2.13) $$\begin{cases} 2\alpha = Ar \\ 2\eta = Br \\ s = Cr \end{cases}$$

where A, B and C are constant. The first two assumptions of equation (2.13) ensure flatness at the centre and the third makes the scalar field zero at r = 0.

Now with the help of equation (2.13) equations (2.8) to (2.12) provide

(2.14) $$16\pi p = \frac{e^{-Ar}}{r^2}\left[\left(A + B\right)\frac{r}{2} - e^{(A+B)r} + 1\right]$$

(2.15) $$8\pi\rho = \frac{e^{-Ar}}{8r^2}\left[\left(14B - 2A\right)r - \left(3B^2 - A^2 - 2AB + 16\gamma c^2\right)r^2\right.$$

$$+4\left(e^{(A+B)r/2}-1\right)\Big]$$

(2.16) $\psi_1^2 e^{-2\eta} = \frac{1}{2\gamma^2}\left(e^{(A+B)r/2}-1\right)$

$$+\frac{1}{16}(3B^2 - A^2 - 2AB - 16\gamma c^2) - \frac{1}{4r}(A+B)$$

(2.17) $4\pi\sigma = \left(\frac{d}{dr}F^{41} + \left(\frac{2}{r}+A\right)R^{41}\right)e^{\frac{Br}{2}}$

(2.18) $4\pi\beta = 2F\frac{e^{-Ar}}{\gamma r}$

Now we see that e, p, σ, β i.e., mass density, pressure, charge density and scalar charge density all become infinite at the centre r = 0. So there is singularity in these quantities at centre. However if we choose

(2.19) $2\alpha = Ar^2,\ 2\eta = Br^2 \quad s = cr^2$

Then at the centre r = 0, we get

(2.20) $8\pi\rho_0 = \frac{1}{4}(17B - 7a)$

(2.21) $16\pi p_0 = \frac{3}{2}(A+B)$

(2.22) $4\pi\sigma_0 = (3B^2 - A^2 - 2AB + 16\gamma c^2)^{1/2}$

(2.23) $4\pi\beta_0 = 6\frac{C}{\gamma}$

Here if we assume that constant A and B are positive then ρ_0 and p_0 are both positive if

$$B > \frac{7}{17}A$$

Also σ_0 is real if

(2.24) $3B^2 - A^2 - 2AB + 16\gamma c^2 > 0$

The geometric mass of the sphere is

(2.25) $$M = \int_{0}^{a} \int_{\theta=0}^{\pi} \int_{\phi=0}^{2\pi} \rho dv$$

Where dv (the proper elementary volume)

(2.26) $$dv = r^2 e^{2\alpha-\eta} \sin\theta \, d\theta d\phi \, dr.$$

Thus, the geometric mass of the sphere [for the choice equation (19)] is given by

(2.27) $$M = \int_{0}^{a} \left[\frac{1}{4}\left(17B - 7A\right) + \left(\frac{3}{2}A^2 - \frac{9}{2}B^2 + 3AB \right) r^2 \right] \times e^{-\eta} r^2 dr$$

$$+\frac{1}{4}\int_{0}^{a} (2e^{-2\eta}\psi_1^2 + 6\gamma s_1^2) e^{-\eta} r^2 \, dr$$

$$-\mu(a) + \varepsilon(a) + s(a)$$

Thus the mass density, electric charge and scalar charge densities contribute to the geometrical mass of charged fluid sphere as is evident from equation (2.27)

3. Discussion

In this paper we have discussed interior solution for an electric and scalar charged fluid sphere using specific assumptions. Here we have found that the mass density, electric charge and scalar charge densities contribute to the geometrical mass of the charged fluid sphere.

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