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A Study on Oscillation Results for Second Order Matrix Differential Equations for Damping Equation

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Abstract:

In this paper we establish the Wintner oscillation criterion for system by using matrix Riccati type transformation, the generalized averaging pairs and positive linear functional. By using the positive linear functional, including the general means and Riccati technique, some new oscillation criteria are established for the second order matrix differential equations

$$(r(t)P(t)\Omega(X(t))K(X'(t)))' + p(t)R(t)\Omega(X(t))K(X'(t))$$
$$+Q(t)F(X'(t)G(X(t)) = 0; t \geq t_0 > 0$$

The results improve and generalize those given in some previous papers. We study the qualitative behaviour of the positive solutions of second order matrix differential equations with damping equation with initial conditions being positive linear functions, and parameters. More precisely, we investigate existence of positive solutions, boundedness and persistence, and stability analysis of a second-order matrix differential equations with damping equations.

1. Introduction

Consider the second order matrix differential equations of the form

$$(r(t)P(t)\Omega(X(t))K(X'(t)))' + p(t)R(t)\Omega(X(t))K(X'(t))$$
$$+Q(t)F(X'(t))G(X(t)) = 0; t \geq t_0 > 0 \qquad (1.1)$$

where $t_0 \geq 0$ and r, p, P, ψ, K, R, Q and G satisfy the following conditions :

(1) $r \in C^1([t_0\infty);(0,\infty)), p \in C([t_0,\infty);(-\infty,\infty));$

(2) $P(t) = P^T(t) > 0, Q(t) \geq 0, R(t) = R^T(t) > 0$ for $t \geq t_0$, P, Q and R are n × n matrices real valued continuous functions on the interval $[t_0,\infty)$, and P(t) and R(t) are commutative. By A^T we mean the transpose of the matrix. A;

(3) $\psi, K, G, F \in C^1(R^{n^2}, R^{n^2})$, and $\psi^{-1}(X(t)), K^{-1}(X'(t))$ and $G^{-1}(X(t))$

exist for all $X \neq 0$ and $F(X') \geq 0$ for all $X \neq 0$.

We now denote by M the linear space of n × n real matrices, In $\in$M the identity matrix and S the subspace of all symmetric matrices in M. For any A, B, C $\in$ S, we write A $\geq$ B to mean that A – B $\geq$ 0, that is, A – B is positive semi-definite and A > B to mean that A – B > 0, that is A – B is positive definite. If A and B are positive definite matrices, then $B^{-1} - A^{-1}$ is positive definite matrix. Note that A ± B and A' are also symmetric matrices, where$'$ denotes the first derivative. We will use some properties of this ordering, that is, A > B implies that $C^TAC > C^TBC$.

We call a matrix function solution $X(t) \in C^2((t_0;1);R^{n^2})$ of (1.1) is prepared nontrivial if det $X(t) \neq 0$ for at least one $t \in [t_0;1)$ and X(t) satisfies the equation

$$G^T(X(t))P(t)\psi(X(t))K(X'(t))) - (K(X'(t)))^T \psi^T(X(t))P(t)G(X(t) \equiv 0) \quad (1.2)$$

$$G^T(X(t))P(t)\psi(X(t))K(X'(t))) - (K(X'(t))^T \psi^T(X(t))R(t)G(X(t) \equiv 0 \quad (1.3)$$

and

$$\psi^T(X(t))G'(X(t))X'(t)K^{-1}(X'(t)) - (K^T(X'(t)))^{-1}(X'(t))T(G'(X(t)))^T$$

$$\psi x(t) \equiv 0, \; t \geq t_0 > 0 \quad (1.4)$$

A prepared solution X(t) of (1.1) is called oscillatory if det X(t) has arbitrarily large zeros; otherwise, it is called non oscillatory.

For n = 1, oscillatory and non oscillatory behaviour of solutions for various classes of second-order differential equations have been widely discussed in the literature (see, for example, [3, 5, 10, 13-16, 19-21, 23, 24, 27, 34, 35, 36, 37-39, 42-48] and references quoted there in). In the absence of damping, there is great number of papers [24, 29, 30, 33, 34, 38, 40, 41, 46] dealing with particular cases of equation (1.1) for n = 1 such as the linear equations.

$$x''(t)+q(t)x(t)=0 \qquad (1.5)$$

$$(r(t)x'(t))'+q(t)x(t)=0 \qquad (1.6)$$

and the nonlinear equations

$$(r(t)x'(t))'+q(t)1(x(t))=0 \qquad (1.7)$$

$$(r(t)(x(t))x'(t))'+q(t)g(x)(t))=0 \qquad (1.8)$$

In 2000, the second order nonlinear differential equation

$$x''(t)+q(t)f(x(t))g(x'(t))=0 \qquad (1.9)$$

has been studied by Li and Agarwal [29]. Motivated by the ideas of Li [28] and Rogovchenko [43], they obtained new oscillation criteria for oscillation by using a generalized Riccati technique.

For n = 1, oscillation of nonlinear differential equations with a linear damping term of the form (1.1) that is,

$$(r(t)x'(t))'+p(t)x'(t)+q(t)f(x(t))=0 \qquad (1.10)$$

has been addressed in the monograph of Agarwal et. al. [2] and papers by Elabbasy et al. [11]. Grace and Lalli [19], Hao and Lu [22], Kirane and Rogovchenko [25], Li and Agarwal [30], Li et al. [32], Rogovchenko [39], Rogovchenko and Tuncay [41], [42], to mention a few, whereas oscillation criteria for the general equation

$$(r(t)\psi(x(t)x'(t))'+p(t)x'(t)+q(t)+q(t)f(x(t))=0 \qquad (1.11)$$

were suggested, for instance, Grace [13, 15], Grace and Lalli [17, 18] and Monojlovic [35], Rogovchenko and Tuncay [40].

In 2014, the oscillation of a second-order nonlinear differential equation with damping.

$$(r(t)(x'))^{\gamma})' + p(t)(x'(t))^{\gamma} + q(t)f(x(t)) = 0 \qquad (1.12)$$

studied by Li et. al. [31] for $\gamma \geq 1$ is a ratio of odd positive integers. They extended the results of Rogovchenko and Tuncay [41].

Recently, there has been an increasing interest in studying oscillation and non oscillation of different classes of differential equations. For n $\geq$ 1, p-Laplace equations have some applications in continuum mechanics as seen from [3, 4]. Zhang et al. [34] concerned with the problem of oscillation and asymptotic behavior of a higher-order delay damped differential equation with p-Laplacian like operators

$$(a(t)|x^{(n-1)}t|^{p-2}x^{(n-1)}t' + (r(t)|x^{(n-1)}t|^{p-2}x^{(n-1)}t + q(t)$$

$$|x^{(n-1)}g(t)|^{p-2}x(g(t)) = 0 \qquad (1.13)$$

where p > 1 is a real number. They obtained oscillation results by using the integral averaging technique for Eq. (1.13). Also, they obtained asymptotic results by using the comparison technique. In the special case when p = 2 and n = 2, Eq. (1.13) reduces to Eq. (1.10).

In recent years, there has been an increasing interest in studying oscillatory behaviour of solutions to various classes of dynamic equations on time scales. In particular, oscillation of dynamic equations with demping has become an important area of research due to the fact such equations arise in many real life problems; see e.g. [6-8, 44] and the references cited therein. In 2015, the following dynamic equations with damping studied by several authors.

$$(a(x^{\Delta})^{\gamma})^{\Delta}(t) + p(t)((x^{\Delta})^{\gamma})(t) + q(t)x^{\gamma}(\delta(t)) = 0 \qquad (1.14)$$

$$(r(x^{\Delta})^{\gamma})^{\Delta}(t) + p(t)\left((x^{\Delta^{\sigma}})^{\gamma}\right)(t) + q(t)f(x)(\tau(t) = 0 \qquad (1.15)$$

And

$$(a(t)\psi x(t)(x^{\Delta})(t))^{\Delta} + p(t)\left((x^{\Delta\sigma})^{\gamma}\right)(t) + q(t)f(x^{\sigma}(t)) = 0 \qquad (1.16)$$

where $t \in [t_0;\infty)T := [t_0;\infty) \cap T, T$ is a time scale which is unbounded above. In the special case when T = R, equations (1.14) and (1.16) reduce to equations (1.10) and (1.11). Agarwal et al. [1] studied for Eq. (1.14). They obtained new oscillation criteria for Eq. (1.14) by using the generalized Riccati transformation technique.

Bohner and Li [9] established a new Kamenev-type theorem for Eq. (1.15) by using the generalized Riccati transformation technique. Wang et al. [51] considered with Eq. (1.16). They obtained several suient conditions for the oscillation of solutions for Eq. (1.16) by using the Riccati transformation and integral averaging technique.

For n > 1, self-adjoint second order matrix differential systems arise in many dynamical problems studied by several authors (e.g., see [12, 26] and references quoted therein). In the special cases of (1.1), Eq. (1.1) reduces to the following second-order matrix differential equations :

$$(P)(t)X'(t))' + Q(t)X(t) = 0; \qquad t \geq t_0 > 0 \qquad (1.17)$$

$$(P(t)X'(t))' + p(t)P(t)X'(t) + Q(t)X(t) = 0 \qquad t \geq t_0 > 0 \qquad (1.18)$$

$$(P)(t)X'(t))' + R(t)X'(t) + Q(t)X(t) = 0; \qquad t \geq t_0 > 0 \qquad (1.19)$$

$$(r(t)X'(t))' + p(t)X'(t) + Q(t)G(X(t)) = 0; \quad t \geq t_0 > 0 \qquad (1.20)$$

$$(r(t)X'(t))' + p(t)X'(t) + Q(t)F(X'(t))G(X(t)) = 0; \quad t \geq t_0 > 0 \qquad (1.21)$$

and

$$(r(t)P(t)X'(t))' + p(t)P(t)X'(t) + Q(t)F(X'(t)G(X(t)) = 0 \quad t \geq t_0 > 0 \quad (1.22)$$

The oscillatory solution properties of equations (1.1) and (1.17) - (1.22) are important in the mechanical systems associated with (1.1)

Therefore, such properties have been studied quite extensively (see [12, 29, 30] and references quoted therein). Eq. (1.20) the results of Li and Agarwal [30] for scalar cases. In 2005 and 2006, Sun and Meng [47, 48] established some oscillation criteria by using the positive linear functional for (1.19). Also, in 2008, motivated by [26], Xu and Zhu [53) obtained several Wintner-type oscillation criteria for system (1.19). These results improved and generalized most known results. In 2013, by using a matrix Riccati type transformation and matrix inequalities, Shi et al. [45] obtained some new oscillation criteria for the second order nonlinear matrix differential systems with damped term

$$(P(t)X'(t))' + R(t)X'(t) + F(t, X(t), X'(t)) = 0 \quad t \geq t_0 \tag{1.23}$$

Motivated by the idea of Li and Agarwal [29], in this paper we establish the Wintner type oscillation criterion for system of (1.1) by using matrix Riccati type transformation, the generalized averaging pairs and positive linear functionals, we establish the Wintner type oscillation criterion for system of (1.1).

In section 2 several definitions and Lemmas are given. Section 3 establish Wintner type oscillation criteria. Finally, in section 4 several examples that dwell upon the sharpness of our results are presented.

2. Definitions and Lemmas

Definition 2.1. Denote by M the linear space of a real matrices, by $I_n \in M$ the identity matrix and S the subspace of all symmetric matrices in M. A linear functional L on M is said to be "positive" if L (A) > 0 for any $A \in S$ and A > 0.

Definition 2.2. A pair of real-valued functions (f, g) defined on $[t_0, \infty)$ is called an averaging pair if

(i) f is nonnegative and locally integrable on $[t_0, \infty)$ satisfying $\int_{t_0}^{\infty} f(s)ds \neq 0$;

(ii) g > 0 is absolutely continuous on every compact subinterval of $[t_0, \infty)$; and

(iii) for $0 \leq k < 1$,

$$\lim_{t\to\infty} \int_{t_0}^{t} f(s) \left[\left(\int_{t_0}^{s} g(u) f^2(u) du \right)^{-1} \left(\int_{t_0}^{s} f(u) du \right)^{k} \right] ds = \infty$$

Definition 2.3. Let L be a positive linear functional and B = B(t) a real valued matrix function which is invertible for each $t \in [t_0, \infty)$. A quartet of real-valued functions (f, g, L, B) defined on $[t_0, \infty)$ is a generalized averaging quartet if the conditions (i) and (ii) in Definition 2.2 and the following conditions (iii) hold

(iii) for $0 \leq k < 1$,

$$\lim_{t\to\infty}\int_{t_0}^{t} f(s)\left[\left(\int_{t_0}^{s} g(u)f^2(u)L(B(u))du\right)^{-1}\left(\int_{t_0}^{s} f(u)du\right)^{k}\right]ds = \infty$$

Lemma 2.4

(I) Let conditions in Definition 2.3 hold then $\int_{t_0}^{\infty} f(s)ds = \infty$

(II) Let $c \in C([t_0, \infty), R)$ and $\int_{t_0}^{\infty} f(s)ds = \infty$; then

$$\lim_{t\to\infty}\left[\left(\int_{t_0}^{\infty} f(s)ds\right)^{-1}\int_{t_0}^{\infty} f(s)c(s)ds\right]ds = \infty$$

Implies

$$\lim_{t\to\infty}\left[\left(\int_{\tau}^{\infty} f(s)ds\right)^{-1}\int_{\tau}^{\infty} f(s)c(s)ds\right]ds = \infty, \quad t \geq \tau \geq t_0$$

Lemma 2.5. [36] Let L be a positive linear functional on M. Then, for any $A; B \in S$, we have

$(L[A^T B])^2 < L[A^T A]L[B^T B]$

Lemma 2.6. Let L be a positive linear functional on M. For any $R \in M, B \in S$ and $B > 0$, then for all $v \in C([t_0, \infty), (0, \infty))$

$$L\left[\frac{1}{v}R^T BR\right] \geq (vL[B^{-1}]^{-1}(L[R])^2$$

Lemma 2.7. Let X(t) be a nontrivial prepared solution of (1.1) and det $X(t) \neq 0$ for $t_0 \geq 0$. Then for all $a \in C^1((t_0,\infty),(0,\infty))$ the matrix function

$$W(t) = a(t)r(t)P(t)\psi X(t))K(X'(t))G^{-1}(X(t)) \qquad (2.1)$$

Satisfies the equation

$$W'(t) = \frac{a'(t)}{a(t)}W(t) - \frac{p(t)}{r(t)}R(t)p^{-1}(t)W(t) - a(t)Q(t)F(X'(t))$$

$$-\frac{W(t)G'(X(t))X'(t)K^{-1}(X'(t)\psi^{-1}(X(t))P^{-1}(t)W(t)}{a(t)r(t)} \qquad (2.2)$$

3. Main Results

In this section, by using matrix Riccati type transformation, the generalized averaging pairs and positive linear functionals, we establish the Wintner type oscillation criterion for system of (1.1)

Theorem 3.1. Assume that all conditions stated in Section 1 are satisfied; suppose for any solution X(t) for (1.1),

$$G'(X(t))X'(t)K^{-1}(X'(t))^{-1}(X(t)) > 0$$

For $t \geq t_0$, and P(t) and R(t) are commutative with $G'(X(t))X'(t)K^{-1}(X'(t))^{-1}(X(t))$.

Suppose further that there exists a function a $\in C^1([t_0,\infty),(0,\infty))$ and a generalized averaging quartet

$$(f, ar, L; P(t)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(X(t)))^{-1}),$$

where L is a positive linear functional on M, satisfying

$$\operatorname*{Lim}_{x\to\infty} L\left[\Xi_{t_0}^{t} J(t_0,t)\right] = \infty \qquad (3.1)$$

and the matrix J defined by

$$J(t_0,t)=\frac{1}{2}(a'(t)r(t)I_n-a(t)p(t)R(t)P^{-1}(t))P(t)\psi(X(t))$$

$$\times K(X'(t))(X'(t))^{-1}\left(G'(x(t))\right)^{-1}$$

$$+\int_{t_1}^{t}\left[a(s)Q(s)F(X'(s))-\frac{(a'(s)r(s)I_n-a(s)p(s)R(s)P^{-1}(s))}{4a(s)r(s)}\times\right.$$

$$P(s)\psi(X(s))K(X'(s))(X'(s))-1GXs-1d(s) \quad (3.2)$$

and $\Xi_{t_0}^{t}:M\to M$ is the linear operator defined by

$$\Xi_{t_0}^{t}U(t)=\left(\int_{t_0}^{t}f(s)ds\right)^{-1}\int_{t_0}^{t}f(s)U(s)ds \quad (3.3)$$

Then every prepared solution of (1.1) is oscillatory on $[t_0,\infty)$.

Proof : Suppose the Theorem 3.1 is not true and X(t) is any nontrivial prepared solution of (1.1) in $[t_1,\infty)$ which is nonoscillatory. Without loss of generality, assume that det $X(t)\neq 0, t\geq t_1\geq t_0$. Then by Lemma 2.7, W(t) is symmetric and satisfies the Riccati equation (2.2).

That is,

$$W'(t)=\frac{a'(t)}{a(t)}W(t)-\frac{p(t)}{r(t)}R(t)P^{-1}(t)W(t)-a(t)Q(t)F(X'(t))$$

$$-\frac{W(t)G'(X(t))X'(t)K^{-1}(X'(t))\psi^{-1}(X(t))P^{-1}(t)W(t)}{a(t)r(t)} \quad (3.4)$$

Integrating both sides of (3.4) from t_1 to t, we obtain

W(t)

= W(t_1)

$$+\int_{t_1}^{t}\left[\frac{a'(t)}{a(t)}W(t)-\frac{p(t)}{r(t)}R(t)P^{-1}(t)W(t)-a(t)Q(t)F(X'(t))\right.$$

$$-\frac{W(t)G'(X(t))X'(t)K^{-1}(X'(t))\psi^{-1}(X(t))P^{-1}(t)W(t)}{a(t)r(t)}\Bigg]ds \qquad (3.5)$$

Now use of previous lemma and integrate them we get which contradicts the fact

$$(f, ar, L, P(t)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(X(t)))^{-1})$$

is a generalized averaging quartet.

Corollary 3.2. If the above conditions hold and

$$G'(X(t))X'(t)K^{-1}(X'(t))\psi^{-1}(X(t))P^{-1}(t) \geq A > 0$$

and

$$F(X'(t)) \geq B > 0 \quad t \in [t_0, \infty)$$

where A, B $\in S$ are constant positive definite matrices, and A is commutative with P(t) and R(t). Suppose further that there exist an averaging pair (f, ar), where $a \in C^1([t_0,\infty),(0,\infty)$ and L is a positive linear functional on M satisfying (3.1), where

$$J(t_0, t) = \frac{1}{2}\left(a'(t)r(t)I_n - a(t)p(t)R(t)P^{-1}(t)\right)A^{-1}$$

$$+\int_{t_1}^{t}\left[a(s)Q(s)F(X'(s)) - \frac{(a'(s)r(s)I_n - a(s)p(s)R(s)P^{-1}(s))^2}{4a(s)r(s)}A^{-1}\right]d(s)$$

and $\Xi_{t_0}^{t} : S \to S$ is the linear operator defined by (3.3). Then any prepared solution of (1.1) is oscillatory on $[t_0, \infty)$.

Remark 3.3. Theorem 3.1 and Corollary 3.2 are improvement and generalize of theorem 3.1 and Corollary 3.1 by Yang [56]. In fact, Theorem 3.1 in [56] is not applicable if we choose such that

$$\lim_{t\to\infty}\int_{t_0}^{t}\frac{ds}{a(s)r(s)L[P(s)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(X(t)))^{-1}]} < \infty$$

or $\quad P(t) \neq R(t)$

Remark 3.4. Theorem 3.1 is improvement and generalize of Theorem 3.1 by Xu and Zhu [53]. In fact. Theorem 3.1 in [53] is not applicable if we choose such that

$$\lim_{t\to\infty}\int_{t_0}^{t}\frac{ds}{a(s)r(s)L[P(s)\psi(X(t))K(X'(t))(X'(t))^{-1}(G'(X(t)))^{-1}]}<\infty$$

Remark 3.5. Theorem 3.1 and Corollary 3.2 are improved and generalize of Theorem 3.1 and Corollary 3.1 by Yang and Tang [59]. In fact, Theorem 3.1 in [57] is not applicable if we choose such that P(t) $\neq$ R(t). But when P(t) = R(t), ψ (X(t)) = I_n and $K(X'(t)) = X'(t)$ in Theorem 3.1 and Corollary 3.2 give Theorem 3.1 and Corollary 3.2 in [57], respectively. Also, when $G'(X(t)) > 0$ and P(t) > 0 in Theorem 3.1 [57], the product of these positive definite matrices is not necessarily positive definite.

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