International Research Journal of Mathematics, Engineering and IT
ISSN: (2349-0322)
Association of Academic Researchers and Faculties (AARF)
Impact Factor- 7.132 Volume 12, Issue 08, August 2025
Website- www.aarf.asia, Email : editoraarf@gmail.com

The Study of Dihedral group and its isomorphism with the Symmetric group

Akshay Kumar Vyas[1] Ram Milan Singh[2] Amar Deep Singh[3]
[1,3]Department Of Mathematics, Government Motilal Vigyan Mahavidhyalaya, Bhopal, India
[2]Department Of Mathematics, Institute for Excellence in Higher education, Bhopal, India
Email- vyasakshaykumar@gmail.com

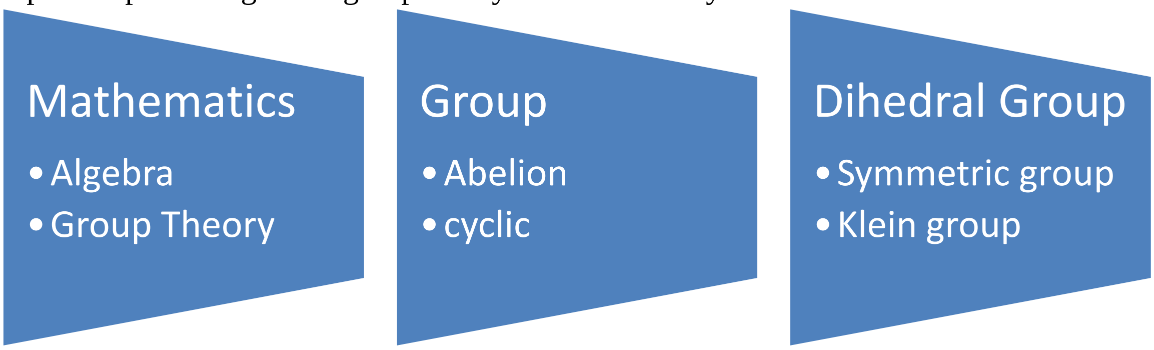
Abstract

The algebraic study of Dihedral group of Algebra is the study by which it is relating to abelion group, cyclic group, normal group. We develop a comprehensive framework integrating advanced geometric invariants, rigorous theorems, and computational validations, supported by vibrant, multi-colored visualizations. The various application of Dihedral group can be easily understood by my paper. Our findings reveal intricate symmetry distributions across 3D volumes, offering profound implications for fields such as computational topology, robotic kinematics, and materials science. By synthesizing algebraic rigor with spatial intuition, this work establishes a transformative paradigm for symmetry studies, poised to inspire groundbreaking interdisciplinary research.

Keywords: Applications, Symmetric group, computer, klein group, bioinformatics; astrophysics, symmetry analysis.

1.Introduction

Mathematics is the queen of science. The algebraic study of some dihedral group mainly deals with geometry of science. Dihedral group is group just like abelion group, cyclic group, normal group. The dihedral group has significance impotance in Algebra, Graph theory and various other coding. It has significance importance in mathematics. Dihedral group mainly deals with rotation and reflection. It has importance in pictorical representation. Dihedral group is an important part of algebraic group theory. It is denoted by Dn.



It is written in the form of equation and we can use different numbers to form different groups. It can have different representation. My research paper usually deals with symmetric group, Dihedral group and klein group and their relation in the higher mathematics. The dihedral group has rotation and reflection inherit in it. It act as two n of the group. So the order of the group is denoted by

$$o(D_n)=2n$$

here ‘n’ is Natural number. The order of the group has a wide importance in the field of mathematics and algebra. Algebra is incomplete without giving the order of the group. The order of the group not only defines the group but also have a vast significance in defining a group. My research article has relation of dihedral group with the other groups such as klein group, symmetric group and various other groups. This advanced research work has application of symmetric group. It is written in the form of factorial which is expressed in the multiple of numbers. Numbers in the form of factorial are also the part of dihedral and symmetric representation. The diagrammatic representation also comes in the property of dihedral group. It had wide scope in cryptography, and coding to define the newly entered value in an hidden codes as well as security codes. The polygons such as pentagon hexagon and various other can be easily explained by dihedral group. It has importance in various fields of graph theory also. A tree with various dimentions can be introduced by it. The dihedral group is also related to klein group in the form of D_2 and K_4. The klein group plays an important role in the advanced abstract algebra. These group are well structured with their definition and works on the basis of their structure. The equation of dihedral group has application in the real world. The dihedral group has well structured outcomes in the form of cayley diagram and charts. These charts help us in representing the dihedral group more accurately. There are different planes to understand the geometry of figures and different groups. My intent is to form the group and follow its property. My goel in this paper is simplify the relation between the two group and simplify it in the form of mathematical illustration. Then i have investigated the mathematical deduction of the paper and tried to prove its algebraic relation with the other group. The group is a combination of these properties and I have also included reference in it as the part of my research paper.

2.Background and Methodology

2.1 [1] Group

Let G be non empty set and “o” is any operation then (G,o) is said to be group if it satisfies four axioms

1.**Closure axiom**

$\forall a \in G, \forall b \in G$ such that $a.b \in G$

2.**Associative axiom**

$ao(boc)= (aob)oc \ \forall\ a,b,c \in G$

Mapping composition satisfy associative axiom

3**Identity axiom**

$\forall a \in G$ there exists $e \in G$ such that $aoe = eoa = a \ \forall\ a \in G$

4.**Inverse axiom**

$\forall a \in G$ there exists unique $a^{-1} \in G$ such that $ao\, a^{-1}= a^{-1}oa = e \ \forall\ a \in G$

2.2 [1] Subgroup

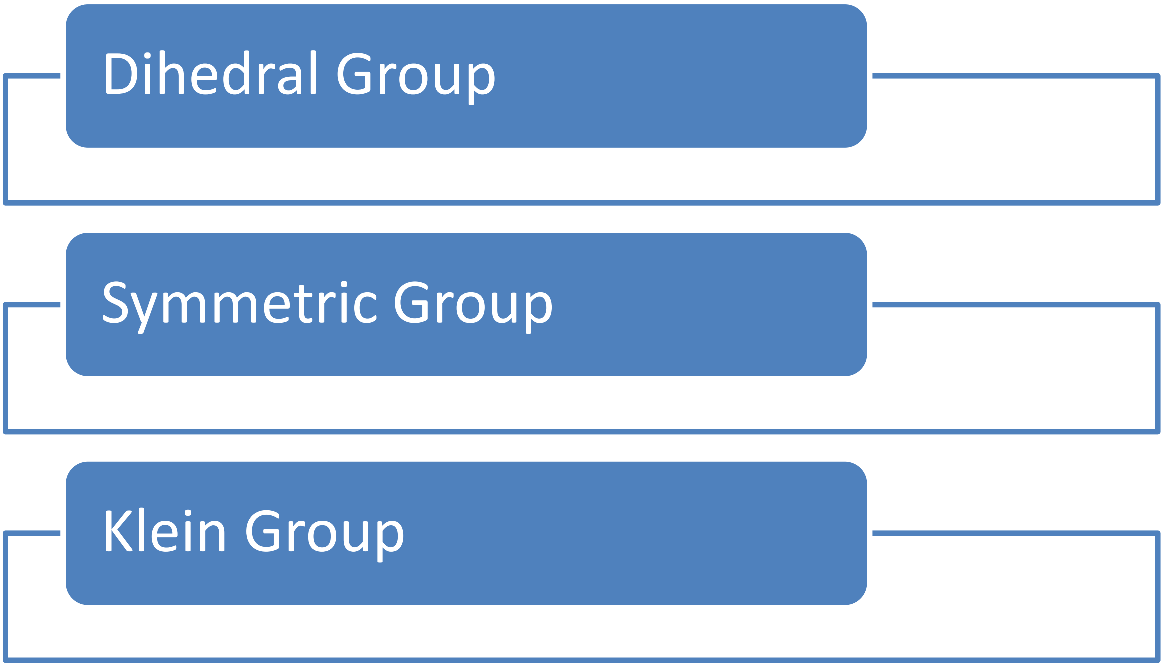
Let $0 \neq H$ is subset of G. H is said to be subgroup of G if H is itself a group with operation of G.

2.3 [1] Lagranges Theoram

If G be a finite group and H is subgroup of G then order of H divides the order of group.

If x does not divides the order of the group then G has no subgroup of order d. The subgroup of a group can be abelian. Subgroup has a important part in field of algebraic mathematics. Lagranges theorem not only proves it in the form of mathematics but also classify it.

3. Results and Discussions



3.1 Relation of Dihedral group and Symmetric Group

The symmetric group is denoted by $S_{n.}$ The order of the group is written as n!. The symmetric group has its own order and its own representation.

Sn is one-one and onto mapping from set containing n elements to itself. Symmetric group has wide importance and application in mathematics. For example

S_3= {1,(1 2), (1 3), (2 3), (1 2 3), (1 3 2)}

o(1)=1

o(1 2)=2

o(1 3)=2

o(2 3)=2

o(1 2 3)=3

o(1 3 2)=3

so S_3 has 1 element of order 1

3 element of order 2

2 element of order 3

The dihedral group $D_3=\{ x^i\ y^j \mid x^2 =e,\ y^3=e,\ xy=y^{-1}x , i=0,1, j=0,1,2\}$

$D_3 =\{R_0, R_{120}, R_{240}, f_{Aa}, f_{Bb}, f_{Cc}\}$

$o(R_0)=1$

$o(R_{120})=3$

$o(R_{240})=3$

$o(f_{Aa})=2$

$o(f_{Bb})=2$

$o(f_{Cc})=2$

so D_3 has has 1 element of order 1

3 element of order 2

2 element of order 3

$D_3 \approx S_3$

So dihedral group D_3 is isomorphic to Symmetric group S_3.

3.2 Relation between dihedral group and klein group

The klein group is defined as

$K_4 = \{e,a,b, ab / a^2 = e, b^2 = e, ab=ba\}$

The dihedral group D_2 is defined as

$D_2 = \{x^i y^j / x^2=e, y^2=e, xy=yx\}$

$K_4 = D_2$ Representation Theory Of is K_4 same as that of D_2

Sol:- $D_2 = \{ x^i y^j | x^2 =e, y^2 =e, xy=y^{-1}x ,i=0,1, j=0,1\}$

given $y^2 = e$

$y\,y = e$

$\Rightarrow y = y^{-1}$

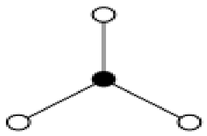
From Equation and we get

$D_2 = \{ x^i y^j | x^2 =e, y^2 =e, xy=y^{-1}x ,i=0,1,j=0,1\}$

$= \{e,x, y, x\,y | x^2 =e , y^2= e, xy = y\,x\} = K_4$

$= K_4 \approx D_2$

$= K_4$ isomorphic to D_2



The diagrammatic representation of D_2

4. Advanced Applications

4.1 Graph theory

The symmetric group can be drawn in graph theory. A tree with four vertices and three adges resembles the group D_3. The group have its own pictorical representation in the form of advanced graph theory. A tree representation in the form of diagram usually takes place by the help of structured cayley diagram and their tool to get data in the form of vertex and edges. The symmetric group resembles the graph theory. Graph theory usually have its diagrammatic way to describe it.[2]

4.2 Computer and Robotics

Today is the era of computer. Symmetric group help us in representing any ordered permutation in coding. The graph, venn diagram have importance in the field of advanced learning and coding. Decorative designs used in pottery, building and floor coverings uses dihedral group of symmetry. Computer fundamentals are incomplete without working in the field of symmetric group it resembles the robotics form of the group and resembles the necessary condition to example the group. Computer is very essential part of the research and without the use of computer and robotics the research won't be too easy and practical. Computer designs are the neccasary elements of research.

4.3 Cryptography

It is a group which hides the information as a security of codes, it act as cryptographic presence of certain codes which are hidden under it. This information can only be traced by the person who have the knowledge about it. Coding has atmost importance in the field of cryptography and essential tools can hidden in the form of security codes. It is also one of the science in mathe matics to form new codes in the form of security keys and do it in the coding of data, its application is highly appreciated in cryptography and other field.

4.4 Algebraic Science

Algebraic science have the advantage in making the group action more secured and accessible to everyone. Science by algebra gives clear explanation about it. Mathematical science is also part of such research work. Algebraic science needs the performance of dihedral group as their combination into the group. Group theory usually deals with the application of algebra, modern algebra and advanced abstract algebra.

4.5 Representation in Three dimentional plane

The three dimensional geometry can be understood by dihedral group and symmetric group. The importance of work done in different planes can be done by figurative and diagrammatic representation. Its has a wide scope in the field of advanced abstract algebra. Various dimention of planes can be understood by dihedral group.[11]

4.6 Mathematical Science

Our study of mathematics is incomplete without learning symmetric group and its applications. The symmetric group has permutation and factorial as a part of their construction. Mathematical science usually deals with the out come of symmetric and dihedral group. They form a balanced form of the group. They are very necessary and important part of algebra. It not only defines a group but also classify the group in the form of symmetry and various other diagrammatic form.

5. Conclusion

The algebraic study of dihedral group has various applications. It also has many relations with symmetric group and klein group. My research paper has shown it in the form of methodology and application. My conclusion is simple and clear about the group. I hope it is helpful in algebraic field of mathematics. The symmetric group resembles the dihedral form and klein form of algebra. My research paper concludes the importance of symmetric and klein group as their original form of algebra.

6. Reference

[1] Gallian, J. A. (2012). Contemporary Abstract Algebra. Cengage Learning

[2] Bondy, J. A. and Murty, U. S. R. (2008). Graph Theory. Springer.

[3] Coxeter, H. S. M. (1973). Regular Polytopes. Dover Publications.

[4] Dummit, D. S. and Foote, R. M. (2004). Abstract Algebra. Wiley.

[5] Fulton, W. and Harris, J. (1991). Representation Theory: A First Course. Springer.

[6] Humphreys, J. E. (1990). Reflection Groups and Coxeter Groups. Cambridge University Press.

[7] Rotman, J. J. (1995). An Introduction to the Theory of Groups. Springer.

[8] Wasserman, L. (2018). Topological data analysis. Annual Review of Statistics and Its Application, 5:501–532.

[9] Armstrong, M. A. (1988). Groups and Symmetry. Springer.

[10] Artin, M. (1991). Algebra. Prentice Hall.

[11] Vyas, Akshay Kumar, Ram Milan Singh, and Amardeep Singh. 2025. “A Transformative Geometric Framework for Dihedral Groups: The Symmetry Density Index in Three-Dimensional Space”. Asian Journal of Pure and Applied Mathematics 7 (1):239-50.

[12] AS Crans, TM Fiore, R Satyendra-Musical actions of dihedral groups- The American mathematical- taylor & francis.

[13] F Ali, M salmon, S Huang; On the commuting graphs of dihedral groups, communications in Algebra ,2016 Taylor & Francis

[14] R Grigorchuk, R Yank; joint spectrum and the infinite dihedral group, proceedings of the steklov institute of mathematics 2017- springer

[15] S Lombardo, AV mikhailov ; Reductions of integrable equations dihedral groups- journal of physics A: Mathematical,2004-ioscience.iop.org

[16] FEB Martinez; Structure of finite dihedral group algebra, finite fields and their applications, 2015- Elsevier

[17] C Xu,R Li;Relation embedding with dihedral group in knowledge graph.

[18] D Handel- on products in the cohomology of dihedral groups; Tohoku Mathematical journal, second series , 1993- jstage.jst.go.jp

[19] K Conrad- Dihedral groups; 2017 math.uconn.edu.

[20] K Conrad Dihedral groups 2; internet online book, 2009- kconrad.math.uconn.edu