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Analytic Calculation of Neutrino Mass from Neutrino Oscillation

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ABSTRACT

The neutrino oscillation experiments provide as with neutrino mass square differences, mixing angles and a possible hierarchy in the neutrino mass spectrum. The main physical goal in future experiment are the determination of the Maxing angle θ_{13}. In particular, the observation of δ is quite interesting for the point of view that δ related to the origin of the matter in the universe. One of the most important parameter in neutrino physics is the magnitude of Mixing angle θ_{13} and CP Phase δ. The oscillation data also suggest that the neutrinos may belong to either a normal hierarchy $(m_1 < m_2 < m_3)$ or an inverted hierarchy $(m_3 < m_1 < m_2)$. The data do not exclude the possibility that the mass of the light neutrino could be much larger than $\sqrt{\Delta_{31}}$, which would imply the possible existence of a quasi-degenerate neutrino mass spectrum $(m_1 \approx m_2 \approx m_3)$. On the other hand, the actual mass of neutrinos cannot be extracted from these data, only the study of tritium single β decay and nuclear neutrino-less double beta decay together can provide sharpest limit on the mass and nature of neutrinos.

Key words:- eigenvalues, neutrino masses, neutrino-oscillation, state neutrino mixing , mass hierarchy

Introduction

Neutrino mass refers to the mass of neutrinos, which are subatomic particles that have very small, but non-zero, masses. While the Standard Model of particle physics initially treated neutrinos as massless, experimental evidence, particularly the observation of neutrino oscillations, proved that they do indeed have mass. Neutrino is the most enigmatic member of the particle zero. Neutrinos, are extremely difficult to detect because they hardly interact with matter at all only one in a million of the neutrinos traversing earth collide with an atomic nucleus. In 1967, Raymond Davis, using a huge tank of clearing fluid (HCL,) placed deep underground to shield it from cosmic rays, started looking for neutrinos produced by the nuclear reactions in the sun's core. However he could only detect about half of what expected. Other underground detectors found similar neutrino shortages. Bruno pontcarvo in 1957 and independently, Maki, Nakagawa and Sakata in 1962, proposed that neutrinos might change from one flavor to another, a phenomenon called neutrino oscillation (oscillation) [Ponteceorvo (1957), Maki et al. (1962)]. Gradually, physicists started to suspect that oscillation could explain the shortfall of neutrinos detected from the sun. The detectors could capture only one type of neutrino, and neutrino during their journey from the sun changed flavor and escaped detection. According to the SM, neutrinos should have zero mass. However quantum theory requires that if neutrinos change flavor they must have mass. Several experiments with atmospheric neutrinos and so called long base line experiments have confirmed that neutrinos change flavor and so have mass.

Solar neutrinos originate from the nuclear fusion powering the sun and other stars. The mechanism by which star's, especially our sun. generate their prodigious quantities of energy has long been of interest to scientists. In 1965 scientists began operating a detector that would detect neutrinos from the interior of the sun and thereby confirm that the sun's emitted energy is derived from the fusion of hydrogen into helium. The neutrinos were observed but at a rate about 1/3 of that predicted by the Standard Solar Model. The oscillation between neutrino families could help to explain this neutrino deficit observed in the solar neutrino flux and could be a good experimental tagging of the fact that neutrinos are massive.

The first observation of electron neutrino deficit was done in Chlorine experiment at Homestake mine. Three additional experiments namely SAGE and GALLEX using gallium as a neutrino target, and Kamiokande, using a water-Cerenko detector have also given evidence for solar-neutrino oscillation. The Super-Kamiokande experiment improved the accuracy in solar neutrino studies greatly using the elastic scattering process. The SNO experiment has studied the charged-current and neutral-current process in addition to the elastic scattering process, and has shown that the solar neutrinos change their flavors from the electron type to other active types [Ahmad et al. (2002)], With these experiments, the decades-old solar neutrino problem that there is a deficit in the flux of neutrinos from the sun as compared to the predictions of the standard solar model championed by Bahcall and his collaborators [Bahcall et al. (1998)] and by many other groups, appears solved

- ❖ Neutrino is the most enigmatic member of the particle zoo. Neutrinos, are extremely difficult to detect because they hardly interact with matter at all only one in a million of the neutrinos traversing earth collide with an atomic nucleus.
- ❖ The neutrino oscillation experiments provide as with neutrino mass square differences, mixing angles and a possible hierarchy in the neutrino mass spectrum.
- ❖ One of the most important parameter in neutrino physics is the magnitude of mixing angle θ_{13} and CP Phase δ. The oscillation data also suggest that the neutrinos may belong to either a normal hierarchy $(m_1 < m_2 < m_3)$ or an inverted hierarchy $(m_3 < m_1 < m_2)$.

Objective of the present work

- ❖ Neutrino oscillations, which only depend on mass square difference, give no information about the absolute value of the neutrino mass squared eigenvalues. Hence, there are various possibilities of neutrino hierarchy spectrums consistent with solar and atmospheric neutrino oscillation data.
- ❖ The neutrino oscillation experiments provide as with neutrino mass square differences, mixing angles and a possible hierarchy in the neutrino mass spectrum.
- ❖ The present work we have attempted to present a picture of neutrino mass spectrum in the case of normal, inverted and almost degenerate hierarchy of neutrino masses by taking some specific choices of effective mass of neutrino.

(The Neutrino Mixing)

- ➢ The experiments with solar, atmospheric, reactor and accelerator neutrinos have provided compelling evidence for oscillation of neutrinos caused by nonzero neutrino masses and neutrino mixing. The data imply the existence of 3- neutrino mixing in vacuum. Below we present the basic 2 neutrino and 3 neutrino mixing schemes briefly.

The Two Neutrino Mixing

- If there are two neutrino flavor eigenstates

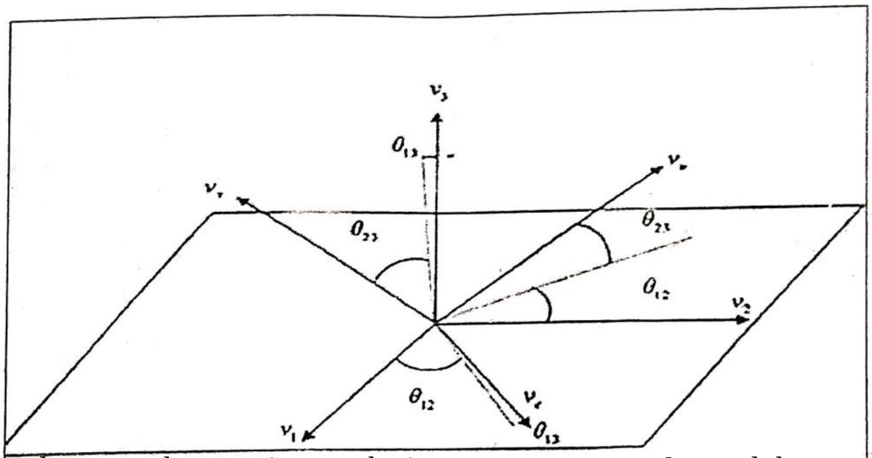
$$\nu_{t_4} = \nu_1 \cos\theta + \nu_2 \sin\theta$$

$$\nu_{t_\phi} = \nu_1 \sin\theta + \nu_2 \cos\theta$$

So that the mixing matrix U given by $\begin{pmatrix} \cos\theta \ \sin\theta \\ -\sin\theta \cos\theta \end{pmatrix}$ is a 2x2 rotation matrix.

Three state neutrino mixing

The minimal neutrino sector required to account for the atmospheric and solar neutrino oscillation data consists of three light physical neutrinos with left-handed flavor eigenstates ν_e, ν_μ *and* ν_τ defined to be those states that share the same doublet as the charged lepton mass eigenstates $e,\ \mu$ *and* τ.



The relation between the neutrino weak eigenstates ν_e, ν_μ *and* ν_τ and the mass Eigenstates ν_1, ν_2 *and* ν_3 in terms of the mixing angles θ_{12}, θ_{13} *and* θ_{23}.

The lepton mixing matrix is summarized in shown below:-

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{23} & S_{23} \\ 0 & -S_{23} & C_{23} \end{pmatrix} \begin{pmatrix} C_{13} & 0 & S_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -S_{13}e^{i\delta} & 0 & C_{13} \end{pmatrix} \begin{pmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{12}^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmosphere	Reactor	Solar	Majorana

The lepton mixing matrix, factorized into matrix product of four matrices.
The phase $\alpha_{1,2}$ are called Majorana phases since they are only present if the neutrino mass is Majorana.

(Effective Majorona Neutrino mass $<M_\nu>$ and Neutrino mass hierarchy)

In the presence of three flavor neutrino mixing the electron neutrino is combination of mass eigenstate, ν_i with eigenvalue m_i.

$$v_e = \sum U_{ej} v_{ij} \ \ i = 1,2,3$$

Here U_{ej} are the elements of the mixing matrix, which relates the flavor states the mass eigenstates.

The effective Majorana neutrino mass $< m_v >$ is $|m_{ee}| = |\sum \ |U_{ej}|^2 e^{i\phi j} m_j|$

Where $$= |c_{13}^2 c_{12}^2 m_1 + c_{13}^2 s_{12}^2 e^{\phi 2} m_2 + s_{13}^2 e^{i\phi_3} m_3|$$

$|U_{ej}|, j - 1,2,3$ are the absolute values of the elements of the first row of neutrino mixing matrix.

Evaluation of effective neutrino mass $[m_v]$

Assuming three state neutrino mixing, the mixing matrix U_{ei} is given by

$$U_{ei} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13} \\ -c_{12}s_{23}s_{13} - c_{23}s_{12} - & s_{12}s_{23}s_{13} + c_{23}c_{12} & s_{23}c_{13} \\ -c_{23}s_{13}c_{12} + s_{23}s_{12} & -c_{23}s_{13}s_{12} - s_{23}c_{12} & c_{23}c_{13} \end{pmatrix}$$

Where, in the above matrix, $c_{ij} = \cos\theta_{ij}$ *and* $s_{ij} = \sin\theta_{ij}$. The allowed range of the angles is $0 \le \theta_{ij} \le \pi / 2$.

The effective mass of neutrino $[m_v]$ is given by element of Ist row of neutrino mixing matrix U_{ei} defined above.

$$|\langle m_v \rangle| \equiv |U_{e1}^L|^2 m_1 + |U_{e2}^L|^2 e^{i\varphi 2} m_2 + |U_{e3}^L|^2 e^{i\theta_3} m_3 \ |$$

$$= |c_3^2 \ c_2^2 \ m_1 + c_3^2 s_2^2 \ e^{i\theta_2} m_2 + s_3^2 e^{i\varphi_3} m_3|$$

where $e^{i\theta_2}, e^{i\theta_3}$ are Majorana phases.

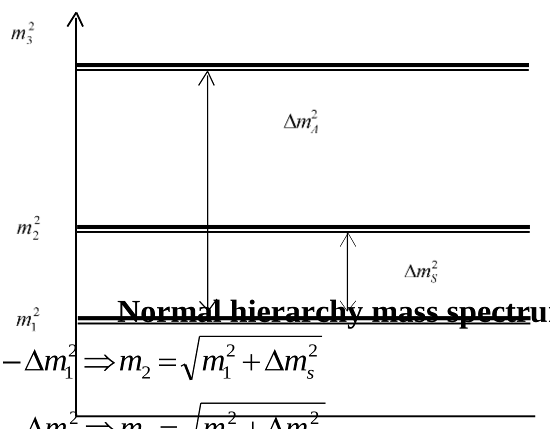
$$= A^2 m_1^2 + B^2 m_2^2 + C^2 m_3^2 + 2ABm_1 m_2 \ (\cos\varphi_2) + 2ACm_1 m_3 (\cos\varphi_3) + 2BCm_3 m_2 \cos(\varphi_2 - \varphi_3)$$

Can be solved for the following three types of neutrino mass hierarchy-

Normal mass hierarchy

This corresponds to the case

$$m_1 << m_2 < m_3.$$



Normal hierarchy mass spectrum

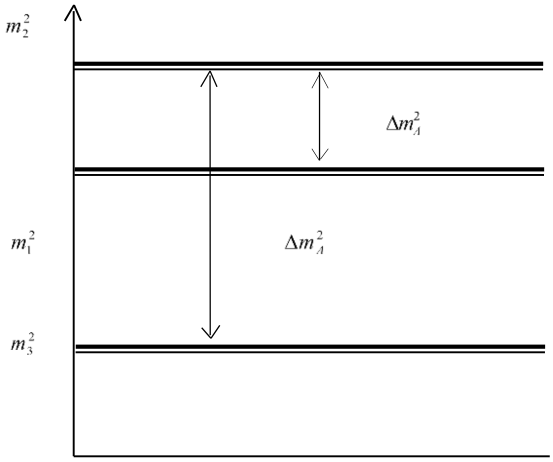
$$\Delta m_s^2 = \Delta m_{21}^2 = \Delta m_2^2 - \Delta m_1^2 \Rightarrow m_2 = \sqrt{m_1^2 + \Delta m_s^2} \quad (3.1)$$

$$\Delta m_A^2 = \Delta m_{31}^2 = \Delta m_3^2 - \Delta m_1^2 \Rightarrow m_3 = \sqrt{m_1^2 + \Delta m_A^2} \quad (3.2)$$

Inverted mass hierarchy

This corresponds to the case

$$m_3 << m_1 < m_2$$



Inverted hierarchy mass spectrum

$$\Delta m_s^2 = \Delta m_{21}^2 = \Delta m_2^2 - \Delta m_1^2 \Rightarrow m_2 = \sqrt{m_1^2 + \Delta m_s^2} \qquad (3.3)$$

$$\Delta m_A^2 = \Delta m_{32}^2 = \Delta m_3^2 - \Delta m_1^2 \Rightarrow m_3 = \sqrt{m_1^2 + \Delta m_s^2 - \Delta m_A^2} \qquad (3.4)$$

Almost degenerate [AD] mass hierarchy

for this type of mass hierarchy, we have

$$m_1 = m_2 = m_3$$

$$\langle m_v \rangle^2 = A^2 m_1^2 + B^2 m_1^2 + c^2 m_1^2 + 2ABm_1^2 \cos\varphi_2 + 2ACm_1^2 \cos\varphi_3 + 2BCm_1^2 \cos(\varphi_2 - \varphi_3)$$
$$= m_1^2 \left[A^2 + B^2 + C^2 + 2AB\cos\varphi_2 + 2CA\cos\varphi_3 + 2BC\cos(\varphi_2 - \varphi_3)\right] \qquad (3.5)$$

Evaluation of neutrino mass spectrum

We can calculate mass eigen-states $m_i\ (i = 1,2,3)$ for the above three mass hierarchies and hence can estimate the neutrino mass spectrum as well.

Normal mass spectrum

$$\langle m_v \rangle^2 - m_1^2 \left(A^2 + B^2 + C^2\right) - B^2 \Delta m_s^2 - c^2 \Delta m_A^2 - 2m_1 AB\cos\varphi_2 \left(m_1^2 + \Delta m_s^2\right)^{1/2} - 2m_1 x$$
$$AC\cos\varphi_3 \left(m_1^2 + \Delta m_A^2\right)^{1/2} = 2BC\left(m_1^2 + \Delta m_s^2\right)^{1/2} \left(m_1^2 + \Delta m_A^2\right)^{1/2} \cos(\varphi_2 - \varphi_3)$$

$$Let\ \ \alpha = \left(m_1^2 + \Delta m_s^2\right) \qquad \beta = \left(m_1^2 + \Delta m_A^2\right)$$

$$and\ \ \gamma = \left[\left(\langle m_v \rangle^2 - B^2 \Delta m_s^2 - C^2 \Delta m_A^2 - m_1^2 \left(A^2 + B^2 + C^2\right)\right)\right]$$

So that on solving the above eqn. in terms of $\alpha, \beta\, and\, \gamma,$ we get

$$+m_1^2\left[-4\delta^3\left(A^2+B^2+C^2\right)+32B^4C^4\left(\Delta m_S^4\Delta m_A^2+\Delta m_s^2+\Delta m_A^4\right)\cos^4\left(\varphi_2-\varphi_3\right)-8B^2C^2\cos^2\left(\varphi_2-\varphi_3\right)\right]$$
$$\left[\delta^2\left(\Delta m_A^2+\Delta m_s^2\right)-2\delta\left(A^2+B^2+C^2\right)\Delta m_A^2\Delta m_s^2\right]-32A^2B^4C^2\cos^2\varphi_2\cos^2\left(\varphi_2-\varphi_3\right)$$
$$\Delta m_A^2\Delta m_s^4-32A^2B^2C^4\cos^4\varphi_3\cos^2\left(\varphi_2-\varphi_3\right)\Delta m_s^2\Delta m_A^4-8A^2B^2\cos^2\varphi_2\delta^2\Delta m_S^2$$
$$-8A^2C^2\cos^2\varphi_3\delta^2\Delta m_A^2-64A^2C^2\cos\left(\varphi_2-\varphi_3\right)\cos\varphi_2\cos\varphi_3\,\delta\Delta m_A^2\Delta m_s^2$$
$$+\left[\delta^4+16B^4C^4\Delta m_s^4\Delta m_A^4\cos^4\left(\varphi_2-\varphi_3\right)-8B^2C^2\cos^2\left(\varphi_2-\varphi_3\right)\delta^2\Delta m_A^2\Delta m_s^2\right]=0$$

Inverted Hierarchy mass spectrum

$$\left\langle m_v\right\rangle^2=A^2m_1^2+B^2\left(\Delta m_s^2+m_1^2\right)+C^2\left(m_1^2+\Delta m_s^2-\Delta m_A^2\right)+2Am_1\left(\Delta m_s^2+m_1^2\right)^{1/2}B\left(\cos\varphi_2\right)+$$
$$2Am_1\left(m_1^2+\Delta m_s^2-\Delta m_A^2\right)^{1/2}C\left(\cos\varphi_3\right)+2BC\left(\Delta m_A^2+m_1^2\right)^{1/2}\left(m_1^2+\Delta m_s^2-\Delta m_A^2\right)^{1/2}$$
$$\cos\left(\varphi_2-\varphi_3\right)$$

Let $\alpha=\left(m_1^2+\Delta m_s^2\right)\qquad \beta=\left(m_1^2+\Delta m_s^2-\Delta m_A^2\right)$

$$\Rightarrow\left[\left\{\left\langle m_v\right\rangle^2-B^2\Delta m_s^2-C^2\left(\Delta m_s^2-\Delta m_A^2\right)\right\}-m_1^2\left(A^2+B^2+C^2\right)\right]-2m_1\,AB\cos\varphi_2\left(\alpha\right)^{1/2}$$
$$2m_1\,x\,AC\cos\varphi_3\,(B)^{1/2}=2BC(\alpha)^{1/2}(\beta)^{1/2}\cos(\varphi_2-\varphi_3)$$

Let $\gamma=\left[\left(\left\langle m_v\right\rangle^2-B^2\Delta m_s^2-C^2\left(\Delta m_s^2-\Delta m_A^2\right)-m_1^2\left(A^2+B^2+C^2\right)\right)\right]$

So that above Eqn. simplify to

$$+m_1^2\left[-4\delta^2\left(A^2+B^2+C^2\right)+32B^4C^4\left(2\Delta m_s^6-3\Delta m_s^4\Delta m_A^2-\Delta m_s^2\Delta m_A^4\right)\cos^4\left(\varphi_2-\varphi_3\right)-\right]$$
$$8B^2C^2\cos^2\left(\varphi_2-\varphi_3\right)\left[2\delta^2\Delta m_s^2-\delta^2\Delta m_s^2-2\delta(A^2+B^2+C^2)\Delta m_s^4+2\delta\left(A^2+B^2+C^2\right)\right.$$
$$\left.\Delta m_A^2\Delta m_s^2\right]-32A^2B^2C^2\cos^2\varphi_2\cos^2\left(\varphi_2-\varphi_3\right)\left(\Delta m_s^6-\Delta m_A^2\Delta m_S^4\right)$$
$$-32A^2B^2C^4\cos^2\varphi_3\cos^2\left(\varphi_2-\varphi_3\right)\left(\Delta m_s^6-2\Delta m_s^4m_A^2-\Delta m_s^2\Delta m_A^4\right)$$
$$-8A^2B^2\cos^2\varphi_2\,\delta^2\Delta m_s^2-8A^2C^2\cos^2\varphi_3\left(\delta^2\Delta m_s^2-\delta^2\Delta m_A^2\right)-64A^2B^2C^2$$
$$\Delta m_A^2\Delta m_s^2\left(A^2+B^2+C^2\right)$$

Almost degenerate [AD] hierarchy mass spectrum

$$\left\langle m_v\right\rangle^2=A^2m_1^2+B^2m_1^2+C^2m_1^2+2ABm_1^2\cos\varphi_2+2ACm_1^2\cos\varphi_3+2BCm_1^2\cos\left(\varphi_2-\varphi_3\right)$$
$$=m_1^2\left[A^2+B^2+C^2+2AB\cos\varphi_2+2CA\cos\varphi_3+2BC\cos\left(\varphi_2-\varphi_3\right)\right]$$
$$=m_1^2\left[A^2+B^2+C^2+2AB\cos\varphi_2+2CA\cos\varphi_3\left(\varphi_2-\varphi_3\right)\right]-\left\langle m_v\right\rangle^2=0$$
$$\Rightarrow m_1^2=\frac{\left\langle m_v\right\rangle^2}{A^2+B^2+C^2+2AB\cos\varphi_2+2CA\cos\varphi_3+2BC\cos\left(\varphi_2-\varphi_3\right)}$$

We find the mass eigen value m_i in case of normal hierarchy, inverted hierchy and almost degenerate mass spectrum respectively. The corresponding m_2 and m_3 values can be calculated from the Eqn. (3.1) and (3.2) in case of normal hierarchy and from Eqn. (3.3) and

(3.4) in case of inverted hierarchy mass spectrum. For almost degenerate case, the three mass eigen values are same.

Table 3.1: Neutrino mass eigen value for normal hierarchy mass spectrum.

Mass Hierarchy	m_ν	Mass Eigenstate	Majorana Phases	Majorana Phases
	(eV)	(eV)	$\phi_2 = 0^0, \phi_3\ 180^0$	$\phi_3 = 0^0 . \phi_2\ 180^0$
Normal	0.010	M_1	1.0×10^{-2}	2.62×10^{-2}
		M_2	1.33×10^{-2}	2.77×10^{-2}
		m_3	5.05×10^{-2}	5.60×10^{-2}
	0.009	M_1	8.89×10^{-2}	2.36×10^{-2}
		M_2	1.25×10^{-2}	2.52×10^{-2}
		m_3	5.03×10^{-2}	5.49×10^{-2}
	0.008	M_1	7.76×10^{-3}	2.11×10^{-2}
		M_2	1.17×10^{-2}	2.28×10^{-2}
		m_3	5.01×10^{-2}	5.38×10^{-2}
	0.007	M_1	6.61×10^{-3}	1.86×10^{-2}
		M_2	1.09×10^{-2}	2.05×10^{-2}
		m_3	4.99×10^{-2}	5.29×10^{-2}
	0.006	M_1	5.43×10^{-3}	1.61×10^{-2}
		M_2	1.03×10^{-2}	1.83×10^{-2}
		m_3	4.98×10^{-2}	5.20×10^{-2}
	0.005	M_1	4.21×10^{-3}	1.36×10^{-2}
		M_2	9.70×10^{-3}	1.62×10^{-2}
		m_3	4.97×10^{-2}	5.13×10^{-2}
	0.004	M_1	4.94×10^{-3}	1.13×10^{-2}
		M_2	9.22×10^{-3}	1.42×10^{-2}
		m_3	4.96×10^{-2}	5.08×10^{-2}
	0.003	M_1	1.61×10^{-3}	8.98×10^{-2}
		M_2	8.89×10^{-3}	1.25×10^{-2}
		m_3	4.95×10^{-2}	5.03×10^{-2}
	0.002	M_1	1.87×10^{-4}	6.84×10^{-2}
		M_2	8.74×10^{-3}	1.11×10^{-2}
		m_3	4.95×10^{-2}	4.99×10^{-2}
	0.001	M_1	1.35×10^{-3}	4.85×10^{-2}
		M_2	8.84×10^{-3}	9.99×10^{-2}
		m_3	4.95×10^{-2}	4.97×10^{-2}

Table 3.2: Neutrino mass eigenvalue for inverted hierarchy mass spectrum.

Mass Hierearchy	m_ν	Mass Eigenstate	Majorana Phases	Majorana Phases
	(eV)	(eV)	$\phi_2 = 0^0, \phi_3\ 180^0$	$\phi_3 = 0^0, \phi_2\ 180^0$
inverted	0.010	M_1	1.04×10^{-1}	2.63×10^{-2}
		M_2	1.04×10^{-1}	2.63×10^{-2}
		m_3	9.18×10^{-2}	2.59×10^{-2}
	0.009	M_1	9.33×10^{-2}	2.37×10^{-2}

		M_2	9.38x10^{-2}	2.37x10^{-2}
		m_3	7.98x10^{-2}	2.32x10^{-2}
	0.008	M_1	8.29x10^{-2}	2.11x10^{-2}
		M_2	8.33x10^{-2}	2.11x10^{-2}
		m_3	6.73x10^{-2}	2.05x10^{-2}
	0.007	M_1	7.24x10^{-2}	1.84x10^{-2}
		M_2	7.29x10^{-2}	1.85x10^{-2}
		m_3	5.38x10^{-2}	1.78x10^{-2}
	0.006	M_1	6.18x10^{-2}	1.58x10^{-2}
		M_2	6.24x10^{-2}	1.58x10^{-2}
		m_3	3.85x10^{-2}	1.51x10^{-2}
	0.005	M_1	5.11x10^{-2}	1.32x10^{-2}
		M_2	5.19x10^{-2}	1.32x10^{-2}
		m_3	1.65x10^{-2}	1.23x10^{-2}
	0.004	M_1	4.11x10^{-2}	1.06x10^{-2}
		M_2	4.19x10^{-2}	1.06x10^{-2}
		m_3	2.56x10^{-2}	9.43 x10^{-2}
	0.003	M_1	3.09x10^{-2}	8.01x10^{-2}
		M_2	3.22x10^{-2}	8.10x10^{-2}
		m_3	3.71x10^{-2}	6.38x10^{-2}
	0.002	M_1	2.07x10^{-2}	5.46x10^{-2}
		M_2	2.25x10^{-2}	5.51x10^{-2}
		m_3	4.38x10^{-2}	2.54x10^{-2}
	0.001	M_1	1.01x10^{-2}	2.67x10^{-2}
		M_2	1.34x10^{-2}	2.81x10^{-2}
		m_3	4.73x10^{-2}	4.03x10^{-2}

Table 3.3: Neutrino mass eigenvalue for degenerated mass spectrum.

Mass Hierearchy	m_v	Mass Eigenstate	Majorana Phases	Majorana Phases
	(eV)	(eV)	$\phi_2 = 0^0, \phi_3\ 180^0$	$\phi_3 = 0^0, \phi_2\ 180^0$
inverted	0.010	M_1	1.04x10^{-1}	2.63x10^{-2}
		M_2	1.04x10^{-1}	2.63x10^{-2}
		m_3	9.18x10^{-2}	2.59x10^{-2}
	0.009	M_1	9.33x10^{-2}	2.37x10^{-2}
		M_2	9.38x10^{-2}	2.37x10^{-2}
		m_3	7.98x10^{-2}	2.32x10^{-2}
	0.008	M_1	8.29x10^{-2}	2.11x10^{-2}
		M_2	8.33x10^{-2}	2.11x10^{-2}
		m_3	6.73x10^{-2}	2.05x10^{-2}
	0.007	M_1	7.24x10^{-2}	1.84x10^{-2}
		M_2	7.29x10^{-2}	1.85x10^{-2}
		m_3	5.38x10^{-2}	1.78x10^{-2}
	0.006	M_1	6.18x10^{-2}	1.58x10^{-2}
		M_2	6.24x10^{-2}	1.58x10^{-2}
		m_3	3.85x10^{-2}	1.51x10^{-2}

	0.005	M_1	$5.11x10^{-2}$	$1.32x10^{-2}$
		M_2	$5.19x10^{-2}$	$1.32x10^{-2}$
		m_3	$1.65x10^{-2}$	$1.23x10^{-2}$
	0.004	M_1	$4.11x10^{-2}$	$1.06x10^{-2}$
		M_2	$4.19x10^{-2}$	$1.06x10^{-2}$
		m_3	$2.56x10^{-2}$	$9.43 x10^{-2}$
	0.003	M_1	$3.09x10^{-2}$	$8.01x10^{-2}$
		M_2	$3.22x10^{-2}$	$8.10x10^{-2}$
		m_3	$3.71x10^{-2}$	$6.38x10^{-2}$
	0.002	M_1	$2.07x10^{-2}$	$5.46x10^{-2}$
		M_2	$2.25x10^{-2}$	$5.51x10^{-2}$
		m_3	$4.38x10^{-2}$	$2.54x10^{-2}$
	0.001	M_1	$1.01x10^{-2}$	$2.67x10^{-2}$
		M_2	$1.34x10^{-2}$	$2.81x10^{-2}$
		m_3	$4.73x10^{-2}$	$4.03x10^{-2}$

RESULT & DISCUSSION

- In table 3.1 table 3.2 and table 3.3, we list neutrino mass eigen value m_j in case of different neutrino mass spectrum.
- We have varied m_v from 1-10 meV and 10-100 meV in case of normal and inverted hierarchy respectively. For almost degenerate case, $m_1 \approx m_2 \approx m_3$
- We have set the majorana phases $\phi = 0^0, 180^0$.

CONCLUSION

- We find the mass eigen value m_i in case of normal hierarchy, inverted mass hierarchy and almost degenerate neutrino mass spectrum.
- We have calculated the mass eigen value m_1, m_2 and m_3 for all the three mass spectrum considered by taking some specific choice of effective neutrino mass depending on the type of mass spectrum.
- The ordering of mass states depends on choice of m_v hence precise determination of m_v from single beta decay experiment (Tritium beta decay) and future neutrino-less double beta decay experiments will make the picture of mass spectrum more clear.

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