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On the Structure equation $F^9 + F = 0$

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Abstract: In this paper, we have studied various properties of the F-Structure manifold satisfying $F^9 + F = 0$. Nijenhuis tensor, F- structures and kernel have also been discussed.
Keywords : Differentiable manifold, projection operators, Nijenhuis tensor, metric and kernel.

1. **Introduction** : Let M^n be a differentiable manifold of class C^∞ and F be a (1,1) tensor of class C^∞, satisfying

(1.1) $F^9 + F = 0$

we define operators l and m on M^n by

(1.2) $l = - F^8$, $m = I + F^8$

whereI is the identity operator on M^n

From (1.1) and (1.2), we have

(1.3) $l + m = I$, $l^2 = l$, $m^2 = m$, $lm = ml = 0$

$lF = Fl = F$, $Fm = mF = 0$

Theorem (1.1): Let the (1,1) tensors p and q be defined by

(1.4) $p = m + F^4$, $q = m – F^4$

then p and q are invertible operators satisfying

(1.5) $p^{-1} = q = p^3$, $q^{-1} = p = q^3$, $p^2 = q^2$, $p^2 – p – q + I = 0$

$q^2 - p – q + I = 0$, $pl = -ql = F^4$, $pm = qm = p^2m = q^2m = m$,

$p^2l = -l = q^2l$

Proof : Using (1.2), (1.3) and (1.4), we have

(1.6) $pq = qp = I$ thus

(1.7) $p^{-1} = q$, $q^{-1} = p$

Also using (1.2), (1.3) and (1.4), we get

(1.8) $p^3 = q$, $q^3 = p$.

From (1.7) and (1.8) we have $p^{-1} = q = p^3$, other results follow similarly.

Theorem (1.2): Let (1,1) tensors α and β be defined by

(1.9) $\alpha = l + F^4$, $\beta = l – F^4$, then

(1.10) $\alpha^2 + \beta^2 = 0$, $\alpha^3 + 2\beta = 0 = \beta^3 + 2\alpha$.

Proof : Using (1.2), (1.3) and (1.9), we get

$\alpha^2 = 2F^4$, $\beta^2 = -2F^4$ Thus $\alpha^2 + \beta^2 = 0$.

The other results follow similarly.

Theorem (1.3): Define the (1,1) tensors γ and δ by

(1.11) $\gamma = m + F^8$, $\delta = m - F^8$, then

(1.12) $\gamma^{-1} = \gamma$, and $\delta = I$

Proof : Using (1.2), (1.3) and (1.11), we get

(1.13) $\gamma = m - l$, $\gamma^2 = I$ thus $\gamma^{-1} = \gamma$ and $\delta = m + l = I$

Theorem (1.3) :Define the (1,1) tensorsξ and η by

(1.14) $\xi = m + F$, $\eta = m - F$ then

(1.15) $\xi^n = m + F^n$,$\eta^n = m + (-1)^n F^n$

Proof :$\eta^2 = m + F^2$, $\eta^3 = m - F^3$ etc

Therefore, $\eta^n = m + (-1)^n F^n$

The other results follow similarly.

2. **Nijenhuis tensor** : The Nijenhuis tensors corresponding to the operators F, l and m be defined as

(2.1) $N(X,Y) = [FX,FY] + F^2[X,Y] - F[FX,Y] - F[X,FY]$

(2.2) $N_l(X,Y) = [lX,lY] + l^2[X,Y] - l[lX,Y] - l[X,lY]$

(2.3) $N_m(X,Y) = [mX,mY] + m^2[X,Y] - m[mX,Y] - m[X,mY]$

Theorem (2.1) : Let F,l and m satisfy (1.1) and (1.2), then

(2.4) (i) $N(mX,mY) = F^2[mX,mY]$

(ii) $mN(mX,mY) = 0$

(iii) $N_l (mX,MY) = [mX,mY]$

(iv) $N_m(lX,lY) = m[lX,lY]$

(v) $N_l(lX,lY) = 0$

(vi) $N_m(mX,lY) = 0$

Proof : With proper replacements of X and Y in (2.1), (2.2) and (2.3) and using (1.3), we get the results

3. **Metric Structure** : Let the Riemannian Metric g, be such that

(3.1) $'F(X,Y) = g(FX,Y)$ is skew-symmetric. Then

(3.2) $g(FX,Y) = -g(X,FY)$, and {F,g} is called the metric F-structure.

Theorem (3.1) : On the metric structure F satisfying (1.1), we get

(3.3)$g(F^4X,F^4Y) = - [g(X,Y) - {'m}(X,Y)]$

where

(3.4) $'m(X,Y) = g(mX,Y) = g(X,mY)$.

Proof : From (1.2), (1.3) and (3.2), (3.4)

$$\begin{aligned} g(F^4X,F^4Y) &= g[X,F^8Y] \\ &= g[X,-lY] \\ &= -g[X,(I - m)Y] \\ &= - [g(X,Y) - {'m}(X,Y)] \end{aligned}$$

4. **Kernel** : Let F be a (1,1) tensor, we define

(4.1) Ker(F) = {X : FX = 0}

Theorem (4.1) : For the (1,1) tensor F satisfying (1.1), we have

(4.2) Ker F = Ker F^2 = ……….. = Ker F^9

Proof : Let X ϵ Ker F

⇒ FX = 0

⇒ $F^2X = 0$

⇒ X ϵ Ker F^2

(4.3) Thus, Ker F⊆ Ker F^2

Now let X ϵ Ker F^2

(4.4)⇒ $F^2X = 0$

⇒ $F^3X = 0$

⇒ …

(4.5) $F^9X = 0$, using (1.1) in (4.5)

we have (4.6) FX = 0

⇒Xϵ Ker F Thus

(4.7) Ker F^2⊆Ker F

From (4.3) and (4.7), we get

(4.8) Ker F = Ker F^2

Proceeding similarly, we get (4.2)

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