

Numerical solution of 1-dimensional heat conduction problem using double interpolation method

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Abstract-

Using mathematical techniques and limited distinctions, this piece attempts to identify the configuration of a one-layered Heat condition complete with starting and limit conditions. We used the Bender-Schmidt repeat connection equation to calculate V(x,t) at various cross section regions. We then applied double interpolation to identify the heat condition's organization as a double interpolating polynomial. We then displayed the solution graphically.

Keywords - Heat conduction equation, Bender Schmidt formula, Interpolation, Boundary value problem

1. Introduction-

Our economy will suffer as a result of the Catatumbo neighbourhood's escalating armed conflict. The current scenario, as evidenced in the increasing rates of homelessness, underlines the importance of providing solutions to this problem. It would be intriguing for the citizens of Catatumbo, Colombia, to see government support generated as a result of the advancement of exploration focused on new technological advancements. The investigation of power-free technology is a significant step in this direction, especially in countries participating in an economic crisis. At the time, one source of increasing revenue is the study of various cooling systems and the consequences that these have on better positions.

Iyengar and Manohar (1988) used the fourth-request distinction investigate to arrange Poisson's scenario in barrel-shaped facilities. They extended the technique to solve the heat situation in two-layered structures with polar positions and three-layered configurations with barrel-shaped arrangements. Marwah and Chopra (1992) developed a truly outstanding scientific methodology for determining the transient intensity conduction condition in a onelayered empty compound chamber with time-dependent limit constraints. According to Sabaeian et al. (2008), temperature conveyance research is critical for estimation, performing, and forecasting heated impacts. Ciegis et al. (2010) create and confirm numerical models and mathematical computations for recreating intensity movement in composite materials. Javed (2012) studied about the development of wet or dry intensity. The phrase "dry applications" describes products like brilliant intensity, steaming water bottles, and electric cushions. Wet intensity has been believed to be more penetrating than dry intensity, however this is more likely to be the case because materials covered in water lose heat more slowly than dry ones. The task of applying a limited contrast method in a barrel-shaped organizing structure to solve a three-layered Poisson's condition was taken up by Shiferaw and Mittal (2013). Mori and Romo (2015) applied the restricted distinction technique to a mathematical recreation of 2D convection-dispersion in barrel-shaped facilities. Kafle et al. (2020) inferred the Forward Time

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Central Space Scheme (FTCSS) for the intensity condition. They also used FTCSS to investigate its mathematical arrangement, and they searched into the mathematical solution and logical arrangement for a range of homogeneous materials (for a variety of benefits of diffusivity α). The results of Khatun et al.'s (2020) study on the safety of a one-layered heat condition were reported. Maturi et al. (2020) proposed a mathematical strategy for determining the proper solution for the intensity conduction condition of copper. Because copper is perfectly matched to lead in terms of heat and electrical conductivity, they focused on it. Salehi and Granpayeh (2020) proposed a limited distinction method as an alternative solution for the two-layered Schrodinger condition in polar directions. Meyu and Koriche (2021) provided a basic treatment of the intensity conduction condition was tackled by Tsega et al. (2022). They achieved this by using tube-shaped facilitators to approximate second-request spatial subordinates with five focal contrasts. Furthermore, he discussed the problem of the strength condition.

2. Formulation of the Problem:

0

0

0.08

0.1

In this discussion, we will focus on the following boundary value problem associated with the one-dimensional heat equation. A partial differential equation (PDE) that is frequently defined by may tell you how hot or cold a rod is

$$\frac{\partial V}{\partial t} = K^2 \frac{\partial^2 V}{\partial t^2} \tag{1}$$

Where V(x, t) is the temperature of the pole estimated at position x at time t, and K is the warm diffusivity of the material, which estimates how well the bar can lead heat, Dependent upon the accompanying limit conditions

Dependent up	boll the accord	npanying min	t conditions				
V(0,t)=0						(2)	
V(1,t)=0						(3)	
V(x,0) = si	ηπχ					(4)	
Where $0 \le x$	\leq 1 and <i>t</i> >	0					
For the soluti	on of this pro	blem, Let we	take $K^2 = 1$				
Analytic Solu	tion of the al	pove problem	is				
V(x,t) = sin	$a\pi x e^{K^2\pi^2 t}$						
The interval of	of differencin	g of x as 0.2 i	i.e. $h = 0.2$				
From Bender	Schmidt equ	ation, the time	e span of <i>t</i> as				
$k = \frac{h^2}{1} = \frac{(0.2)}{10}$	$\frac{2}{2}^{2} = 0.02$					(5)	
$\frac{\pi}{2c^2} = \frac{2}{2c^2}$	$\frac{2}{2} = 0.02$	-0.4 - 0.0	()	u _ 1		(5)	
$1 \text{ nus } x_0 = 0,$	$x_1 = 0.2, x_2$	$= 0.4, x_3 = 0$	$x_4 = 0.8, x_4$	$x_5 \equiv 1$			
$t_0 = 0, t_1 = 0$	$0.02, t_2 = 0.0$	$J4, t_3 = 0.06,$	$t_4 = 0.08, t_5$	= 0.1	1 1 .		
We have a tota	al of 25 mesh p	oints after drav	wing straight li	nes parallel to t	the coordinate a	$ax_{1S}(t,x).$	
Х	0	0.2	0.4	0.6	0.8	1	
t							
0	0	0.59	0.95	0.95	0.59	0	
0.02	0	0.475	0.77	0.77	0.475	0	
0.04	0	0.385	0.6225	0.6225	0.385	0	
0.06	0	0.3113	0.504	0.504	0.3113	0	

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0.408

0.204

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0.252

0.33

0.252

0.33

0

0

0.408

0.204

Table-1							
S. No.	V_{1i}	$\Delta^{0+1}V_{1i}$	$\Delta^{0+2}V_{1i}$	$\Delta^{0+3}V_{1i}$	$\Delta^{0+4}V_{1i}$	$\Delta^{0+5}V_{1i}$	
1	0.59	-0.115	0.025	-0.0087	0.1628	-0.788	
2	0.475	-0.09	0.0163	0.1541	-0.6252		
3	0.385	-0.0737	0.1704	-0.4711			
4	0.3113	0.0967	-0.3007				
5	0.408	-0.204					
6	0.204						

Table-2

S. No.	V_{2i}	$\Delta^{0+1}V_{2i}$	$\Delta^{0+2}V_{2i}$	$\Delta^{0+3}V_{2i}$	$\Delta^{0+4}V_{2i}$	$\Delta^{0+5}V_{2i}$
1	0.95	-0.18	0.0325	-0.0035	-0.159	0.785
2	0.77	-0.1475	0.029	-0.1625	0.626	
3	0.6225	-0.1185	-0.1335	0.4635		
4	0.504	-0.252	0.33			
5	0.252	0.078				
6	0.33					

Table-3

S. No.	V_{3i}	$\Delta^{0+1}V_{3i}$	$\Delta^{0+2}V_{3i}$	$\Delta^{0+3}V_{3i}$	$\Delta^{0+4}V_{3i}$	$\Delta^{0+5}V_{3i}$
1	0.95	-0.18	0.0325	-0.0035	-0.159	0.785
2	0.77	-0.1475	0.029	-0.1625	0.626	
3	0.6225	-0.1185	-0.1335	0.4635		
4	0.504	-0.252	0.33			
5	0.252	0.078				
6	0.33					

Table-4

S. No.	V_{4i}	$\Delta^{0+1}V_{4i}$	$\Delta^{0+2}V_{4i}$	$\Delta^{0+3}V_{4i}$	$\Delta^{0+4}V_{4i}$	$\Delta^{0+5}V_{4i}$
1	0.59	-0.115	0.025	-0.0087	0.1628	-0.788
2	0.475	-0.09	0.0163	0.1541	-0.6252	
3	0.385	-0.0737	0.1704	-0.4711		
4	0.3113	0.0967	-0.3007			
5	0.408	-0.204				
6	0.204					

Table-5

Since both the First and the Last Column of Table 1 contain 0, this means that $\Delta^{0+1}V_{00} = \Delta^{0+2}V_{00} = \Delta^{0+3}V_{00} = \Delta^{0+4}V_{00} = \Delta^{0+5}V_{00} = 0$ (6) And $\Delta^{0+1}V_{50} = \Delta^{0+2}V_{50} = \Delta^{0+3}V_{50} = \Delta^{0+4}V_{50} = \Delta^{0+5}V_{50} = 0$ (7) From Table 2, we get $\Delta^{0+1}V_{10} = -0.115, \Delta^{0+2}V_{10} = 0.025, \Delta^{0+3}V_{10} = -0.0087, \Delta^{0+4}V_{10} = 0.1628, \Delta^{0+5}V_{10} = -0.788$ (8) From Table 3,

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 $\Delta^{0+1}V_{20} = -0.18, \Delta^{0+2}V_{20} = 0.0325, \Delta^{0+3}V_{20} = -0.0035, \Delta^{0+4}V_{20} = -0.159, \Delta^{0+5}V_{20} = -0.159, \Delta^{0+5}V_{20} = -0.159, \Delta^{0+5}V_{20} = -0.159, \Delta^{0+5}V_{20} = -0.0035, \Delta^{0+4}V_{20} = -0.0035, \Delta^{0+4}V_{20} = -0.0035, \Delta^{0+5}V_{20} = -0.0035, \Delta$ 0.785 From Table 4, $\Delta^{0+1}V_{30} = -0.18, \Delta^{0+2}V_{30} = 0.0325, \Delta^{0+3}V_{30} = -0.0035, \Delta^{0+4}V_{30} = -0.159, \Delta^{0+5}V_{30} = -0.159, \Delta^{0$ 0.785 (10)From Table 5 $\Delta^{0+1}V_{40} = -0.115, \Delta^{0+2}V_{40} = 0.025, \Delta^{0+3}V_{40} = -0.0087, \Delta^{0+4}V_{40} = 0.1628, \Delta^{0+5}V_{40} = -0.0087, \Delta^{0+4}V_{40} = 0.0087, \Delta^{0+4}V_{40} = -0.0087, \Delta^{0+4}V_{40} = -0.0087,$ -0.788(11)After carrying out the procedure described above for each row in table 1, we have $\Delta^{1+0}V_{00} = 0.59, \Delta^{2+0}V_{00} = -0.23, \Delta^{3+0}V_{00} = -0.13, \Delta^{4+0}V_{00} = 0.13, \Delta^{5+0}V_{00} = 0.$ $\Delta^{1+0}V_{01} = 0.475, \Delta^{2+0}V_{01} = -0.18, \Delta^{3+0}V_{01} = -0.115, \Delta^{4+0}V_{01} = 0.115, \Delta^{5+0}V_{01} = 0.1$ (13) $\Delta^{1+0}V_{02} = 0.385, \Delta^{2+0}V_{02} = -0.1475, \Delta^{3+0}V_{02} = -0.09, \Delta^{4+0}V_{02} = 0.09, \Delta^{5+0}V_{02} =$ (14) $\Delta^{1+0}V_{03} = 0.3113, \Delta^{2+0}V_{03} = -0.1186, \Delta^{3+0}V_{03} = -0.0741, \Delta^{4+0}V_{03} = -0.0$ $0.0741, \Delta^{5+0}V_{03} = 0$ $\Delta^{1+0}V_{04} = 0.408, \Delta^{2+0}V_{04} = -0.564, \Delta^{3+0}V_{04} = 0.72, \Delta^{4+0}V_{04} = -0.72, \Delta^{5+0}V_{04} = 0.408, \Delta^{2+0}V_{04} = -0.564, \Delta^{3+0}V_{04} = 0.72, \Delta^{4+0}V_{04} = -0.72, \Delta^{5+0}V_{04} = 0.408, \Delta^{2+0}V_{04} = -0.564, \Delta^{3+0}V_{04} = -0.72, \Delta^{5+0}V_{04} = -0.$

$$\Delta^{1+0}V_{05} = 0.204, \Delta^{2+0}V_{05} = -0.078, \Delta^{3+0}V_{05} = -0.048, \Delta^{4+0}V_{05} = 0.048, \Delta^{5+0}V_{05} = 0$$
(17)

T1

) The formula for determining the differences between two orders can be expressed generally as
$$\Delta^{m+n}V_{00} = \Delta^{m+0}V_{0n} - n\Delta^{m+0}V_{0n-1} + \frac{n(n-1)}{2!}\Delta^{m+0}V_{0n-2} - \dots + (-1)^{m}\Delta^{m+0}V_{00}$$
(18)
$$\Delta^{n+m}V_{00} = \Delta^{0+n}V_{m0} - m\Delta^{0+n}V_{m-10} + \frac{m(m-1)}{2!}\Delta^{0+n}V_{m-20} - \dots + (-1)^{m}\Delta^{0+n}V_{00}$$
(19)
$$\Delta^{1+1}V_{00} = \Delta^{1+0}V_{01} - \Delta^{1+0}V_{00} = 0.475 - 0.59 = -0.1150$$
(20)
$$\Delta^{1+2}V_{00} = \Delta^{1+0}V_{02} - 2\Delta^{1+0}V_{01} + \Delta^{1+0}V_{00} = 0.385 - 2 \times 0.475 + 0.59 = 0.0250$$
(21)
$$\Delta^{2+1}V_{00} = \Delta^{2+0}V_{01} - \Delta^{2+0}V_{00} = -0.18 - (-0.23) = -0.1150$$
(22)
$$\Delta^{3+1}V_{00} = \Delta^{3+0}V_{01} - \Delta^{3+0}V_{00} = -0.115 - (-0.13) = 0.0150$$
(23)
$$\Delta^{1+3}V_{00} = \Delta^{1+0}V_{03} - 3\Delta^{1+0}V_{02} + 3\Delta^{1+0}V_{01} - \Delta^{1+0}V_{00} = 0.3113 - 3 \times 0.385 + 3 \times 0.475 - 0.59 = -0.0087$$
(24)
$$\Delta^{2+2}V_{00} = \Delta^{2+0}V_{02} - 2\Delta^{2+0}V_{01} + \Delta^{2+0}V_{00} = -0.1475 - 2 \times (-0.18) + (-0.23) = -0.0175$$
$$\Delta^{1+4}V_{00} = \Delta^{1+0}V_{04} - 4\Delta^{1+0}V_{03} + 6\Delta^{1+0}V_{02} - 4\Delta^{1+0}V_{01} + \Delta^{1+0}V_{00}$$
(25)
$$= 0.408 - 4 \times 0.3113 + 6 \times 0.385 - 4 \times 0.475 + 0.59 = 0.1628$$
$$\Delta^{4+1}V_{00} = \Delta^{4+0}V_{01} - \Delta^{4+0}V_{00} = 0.115 - 0.13 = -0.0150$$
(26)

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$$\Delta^{3+2}V_{00} = \Delta^{3+0}V_{02} - 2\Delta^{3+0}V_{01} + \Delta^{3+0}V_{00} = -0.09 - 2 \times (-0.115) + (-0.13) = 0.0100$$

$$\Delta^{2+3}V_{00} = \Delta^{2+0}V_{03} - 3\Delta^{2+0}V_{02} + 3\Delta^{2+0}V_{01} - \Delta^{2+0}V_{00}$$

= -0.1186 - 3 × (-0.1475) + 3 × (-0.18) - (-0.23) = 0.0139
(28)

Interpolating polynomials in two variables up to the difference of the fifth degree requires the following formula:

$$\begin{split} V(x,t) &= \\ V_{00} + \left[\frac{(x-x_0)}{h} \Delta^{1+0} V_{00} + \frac{(t-t_0)}{k} \Delta^{0+1} V_{00} \right] \\ &+ \frac{1}{2!} \left[\frac{(x-x_0)(x-x_1)}{h^2} \Delta^{2+0} V_{00} + \frac{2(x-x_0)(t-t_0)}{hk} V_{00} + \frac{(t-t_0)(t-t_1)}{k^2} \Delta^{0+2} V_{00} \right] \\ &+ \frac{1}{3!} \left[\frac{(x-x_0)(x-x_1)(x-x_2)}{h^3} \Delta^{3+0} V_{00} + \frac{3(x-x_0)(x-x_1)(t-t_0)}{h^2k} \Delta^{2+1} V_{00} + \frac{3(x-x_0)(t-t_0)(t-t_1)}{hk^2} \Delta^{1+2} V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)}{hk^2} \Delta^{0+3} V_{00} \right] \\ &+ \frac{1}{4!} \left[\frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{h^4} \Delta^{4+0} V_{00} + \frac{4(x-x_0)(x-x_1)(x-x_2)(t-t_0)}{h^3k} \Delta^{3+1} V_{00} + \frac{6(x-x_0)(x-x_1)(t-t_0)(t-t_1)}{h^2k^2} \Delta^{2+2} V_{00} + \frac{4(x-x_0)(t-t_0)(t-t_1)(t-t_2)}{hk^3} \Delta^{1+3} V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{hk^3} \Delta^{1+4} V_{00} + \frac{5(x-x_0)(x-x_1)(x-x_2)(x-x_3)(t-t_0)}{h^2k^3} \Delta^{4+1} V_{00} + \frac{10(x-x_0)(x-x_1)(t-t_0)(t-t_1)(t-t_2)(t-t_3)}{h^2k^3} \Delta^{0+5} V_{00} \right] \\ &+ \frac{10(x-x_0)(x-x_1)((x-x_2)(t-t_0)(t-t_1)}{h^3k^2} \Delta^{1+4} V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)(t-t_4)}{h^2k^3} \Delta^{0+5} V_{00} \right] \\ &+ \frac{2(x-x_0)(t-t_1)(t-t_2)(t-t_3)}{hk^4} \Delta^{1+4} V_{00} + \frac{(t-t_0)(t-t_1)(t-t_2)(t-t_3)(t-t_4)}{k^5} \Delta^{0+5} V_{00} \right] \end{split}$$

After changing the values of the various operators in equation (29) and simplifying the equation, we get the following:

 $\begin{aligned} V(x,t) &= 2.95x - 2.875x(x-t-0.22) - x[2.7083(x-0.2)(x-0.4) + 71.875t(x-0.2) - 156.25t(t-0.02)] + [3.3854x(x-0.2)(x-0.4)(x-0.6) + 15.6250xt(x-0.2)(x-0.4) - 273.4375x(x-0.2)(t-0)(t-0.02) - 906.2500xt(t-0.02)(t-0.04)] - [19.5312xt(x-0.2)(x-0.4)(x-0.6) - 260.4167xt(x-0.2)(x-0.4)(t-0.02) - 3619.8xt(x-0.2)(t-0.02)(t-0.04) - 211980x(t-0.02)(t-0.04)(t-0.06)] \end{aligned}$

3. Graphical Solution:



Graph-1: The space-time graph of numerical solutions of heat conduction equation by double interpolation method for m = 25

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Graph-2: The space-time graph of analytic solution of heat conduction equation



Graph-3: Numerical solution of Heat equation in different values of t

5. Closing Comments: This study introduces a mathematical strategy known as the double interpolation method methodology. This method is used to estimate mathematical configurations of key one-layered heat conditions. Currently, just a few lattices in the proposed technique are focused on guaranteeing the required precision. Because the limit circumstances are considered in a way that is natural, this strategy is quite beneficial for tackling limit value problems. The proposed method is also quite simple to implement, and the mathematical results show that it is extremely effective for the mathematical organization of the stated problem. Furthermore, the concept can be used to various situations involving fractional differentials.

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