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## Stability of Streaming Superposed Fluids with Surface Tension in a Porous Medium

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### Abstract :

The present study investigates the hydromagnetic stability of two superposed streaming viscous fluids in the presence of interfacial surface tension and a uniform vertical magnetic field permeating a homogeneous porous medium. The analysis considers a plane interface separating the fluid layers, each characterized by different densities, viscosities, and streaming velocities. By applying linear perturbation theory, the governing magnetohydrodynamic (MHD) equations are formulated and solved to obtain the dispersion relation that determines the conditions for instability. The combined influence of magnetic field strength, porosity parameter, and surface tension on the growth rate of perturbations is examined in detail. Results reveal that the streaming motion exerts a destabilizing influence by suppressing short-wavelength disturbances, whereas viscosity and surface tension suppressed the stability.

**Key Words:** Hydromagnetic stability, magnetic field, surface tension, porosity.

### 1. Introduction

The Kelvin–Helmholtz (K–H) instability, which arises at the interface between two superposed streaming fluids, plays a fundamental role in numerous astrophysical, geophysical, and laboratory phenomena. The K–H instability appears in a variety of physical contexts—for example, when air flows over mercury, when a highly ionized plasma is enveloped by a cooler gas, or when a meteor penetrates the Earth’s atmosphere. A comprehensive discussion of such hydrodynamic and hydromagnetic instabilities was provided by Chandrasekhar (1961) in his classical monograph. Because of their widespread occurrence in nature and technology, these instabilities continue to be an active area of research.

Considerable work has been devoted to the study of both non-streaming superposed fluids (Rayleigh–Taylor instability) and streaming superposed fluids (Kelvin–Helmholtz instability). Jukes (1964) examined the role of finite electrical conductivity and demonstrated that it introduces new and unexpected modes of behavior. Gerwin (1968) investigated the stability of a non-conducting streaming gas over an incompressible conducting liquid. Bhatia (1974) analyzed the influence of viscosity on two incompressible superposed fluids and concluded that viscosity tends to stabilize the system. Further authoritative treatments of hydrodynamic stability were presented by Joseph (1976) and Drazin and Reid (1981).

Subsequent research has extended the Kelvin–Helmholtz analysis to hydrodynamic, hydromagnetic, and plasma environments. Sengar (1984) studied the stability of two gravitating superposed streams under a uniform vertical magnetic field, showing that magnetic resistivity has a destabilizing influence. In recent decades, flows through porous media have gained particular importance, motivated largely by applications in petroleum engineering—especially crude oil recovery from reservoir rocks. The influence of porous material permeability on hydrodynamic and hydromagnetic instabilities has been studied by Vaghela and Chhajlani (1988), Samaria et al. (1990), Sharma and Kumar (1997), and Khan and Bhatia (2003), highlighting its relevance to rock mechanics and heavy-oil recovery processes. Bhatia and Sharma (2003) analyzed the K–H instability of viscous superposed fluids in a vertical magnetic field within a porous medium.

In the absence of a magnetic field, Allah (2002) investigated the stability of two superposed Newtonian fluids through a porous medium, incorporating the effects of surface tension. Kumar and Lal (2005) examined the stability of superposed Rivlin–Ericksen viscoelastic fluids in porous media, while Kumar et al. (2006) analyzed the instability of rotating superposed Walters' B' viscoelastic fluids under similar conditions. More recently, Kumar et al. (2007) considered the role of viscosity in stratified, superposed non-Newtonian fluids. Notably, most studies on Newtonian and non-Newtonian fluids flowing through porous media have neglected the influence of streaming motion.

Given the significant role of surface tension, magnetic fields, and porous structures in practical and natural environments, it is of considerable interest to investigate their combined influence on the instability of streaming, viscous, electrically conducting fluids. The present study addresses the effect of surface tension on the stability of a plane interface separating two streaming, viscous, electrically conducting fluids permeating a porous medium in the presence of a horizontal magnetic field.

The Kelvin–Helmholtz (K–H) discontinuity, which develops at the plane interface between two superposed streaming fluids, plays a crucial role in a wide range of astrophysical, geophysical, and laboratory phenomena. The K–H instability manifests in several physical situations, such as when air is blown over mercury, when a highly ionized hot plasma is surrounded by a relatively cooler gas, or when a meteor enters the Earth's atmosphere. A comprehensive account of such instabilities in hydrodynamic and hydromagnetic systems was presented by Chandrasekhar (1961) in his classical monograph. Owing to their fundamental importance in real physical contexts, the study of hydrodynamic and hydromagnetic instabilities continues to attract extensive research attention.

The problems of non-streaming superposed fluids (Rayleigh–Taylor instability) and Kelvin–Helmholtz instability have been explored from various perspectives by numerous researchers. Jukes (1964) incorporated the effects of finite electrical conductivity and found that this inclusion led to new and unexpected solutions. Gerwin (1968) investigated the stability of a non-conducting streaming gas flowing over an incompressible conducting liquid, while Bhatia (1974) analyzed the influence of viscosity on the stability of two incompressible superposed fluids and concluded that viscosity exerts a stabilizing effect. Further comprehensive treatments of hydrodynamic stability problems were given by Joseph (1976) and Drazin and Reid (1981).

Subsequent investigations have examined the Kelvin–Helmholtz instability in hydrodynamic, hydromagnetic, and plasma environments. Sengar (1984) studied the stability of two gravitating superposed streams in a uniform vertical magnetic field under the influence of magnetic resistivity and found that resistivity has a destabilizing effect. El-Sayeed (2003) studied the impact of viscosity and finite ion Larmor radius on hydrodynamic transverse instability, while Benjamin and Bridges (1997) provided a Hamiltonian formulation and modern reappraisal of the classical K–H problem in hydrodynamics. Allah (1998) considered the combined effects of magnetic field, heat, and mass transfer on the instability of superposed fluids. Further, El-Ansary et al. (2002) analyzed the role of rotation in the hydrodynamic stability of three-layered systems, and Meignin et al. (2001) and Watson et al. (2004) investigated the K–H instability in Hele–Shaw cells and weakly ionized media, respectively.

In recent years, the flow of fluids through porous media has gained significant attention, particularly due to its applications in petroleum engineering, such as the recovery of crude oil from reservoir rocks. The influence of the permeability of porous media on hydrodynamic and hydromagnetic stability problems has been studied by Vaghela and Chhajlani (1988), Samaria et al. (1990), Sharma and Kumar (1997), and Khan and Bhatia (2003), highlighting its significance in rock dynamics and heavy oil recovery.

In the absence of a magnetic field, Allah (2002) examined the stability of superposed Newtonian fluids through a porous medium considering the effects of surface tension. Kumar and Lal (2005) analyzed the stability of two superposed Rivlin–Ericksen viscoelastic fluids through a porous

medium, while Kumar et al. (2006) studied the instability of rotating superposed Walters' B' viscoelastic fluids in similar conditions. More recently, Kumar et al. (2007) explored the effect of viscosity on stratified, superposed non-Newtonian fluids. Notably, most of these studies on Newtonian and non-Newtonian fluids through porous media have neglected the effects of streaming motion.

It is, therefore, of considerable interest to investigate the influence of surface tension on instability of streaming, viscous, electrically conducting fluids through a porous medium in the presence of a magnetic field. Bhatia and Sharma (2003) investigated the Kelvin–Helmholtz instability of superposed viscous fluids in a vertical magnetic field through a porous medium. The present work extends these investigations by examining the effect of surface tension on stability of a plane interface separating two streaming, electrically conducting, viscous fluids through a porous medium in the presence of a horizontal magnetic field.

## 2. Mathematical formulation

We considered the motion of an incompressible, viscous, infinitely electrically conducting fluid of uniform viscosity  $\mu$ , moving with a uniform horizontal velocity  $U = (U_x, U_y, 0)$  through a porous medium in the presence of uniform vertical magnetic field  $H = (0, 0, H)$ .

The relevant linearized perturbation equations are:

$$\frac{\rho}{\varepsilon} \frac{\partial \mathbf{u}}{\partial t} + \frac{\rho}{\varepsilon} (\mathbf{U} \cdot \nabla) \mathbf{u} = -\nabla \delta p + g \delta \rho + (\nabla \times \mathbf{h}) \times \mathbf{H} + \frac{\mu}{\varepsilon} \nabla^2 \mathbf{u} - \frac{\mu}{\lambda} \mathbf{u} + \sum \left[ T_i \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \delta z_i \right] \delta(z - z_2) \quad \dots(1)$$

$$\varepsilon \frac{\partial (\delta \rho)}{\partial t} + (\mathbf{u} \cdot \nabla) \rho + (\mathbf{U} \cdot \nabla) \delta \rho = 0 \quad \dots(2)$$

$$\varepsilon \frac{\partial \mathbf{h}}{\partial t} + (\mathbf{U} \cdot \nabla) \mathbf{h} = (\mathbf{H} \cdot \nabla) \mathbf{u} \quad \dots(3)$$

$$\nabla \cdot \mathbf{u} = 0 \quad \dots(4)$$

$$\nabla \cdot \mathbf{h} = 0 \quad \dots(5)$$

where  $\mathbf{h} (h_x, h_y, h_z)$ ,  $\delta \rho$ , and  $\delta p$  are the perturbations respectively, in magnetic field  $H$ , density  $\rho$ , and pressure  $p$ , resulting from the disturbance, the Darcian velocity  $\mathbf{u} (u, v, w)$ , to the system. In the above equations,  $\mu$  is the coefficient of viscosity,  $\mathbf{g} = (0, 0, -g)$  is the acceleration due to gravity,  $T$  is surface tension,  $\lambda$  is the permeability of the porous medium,  $\varepsilon$  is the medium's porosity and  $\delta(z - z_s)$  denotes Dirac's  $\delta$  function. Analyzing in terms of normal modes, we assumed that the perturbed quantities have the space  $(x, y, z)$  and time  $(t)$  dependence of the form:

$$f(z) \exp(ik_x x + ik_y y + nt) \quad \dots(6)$$

Where  $f(z)$  is some function of  $z$ ;  $k_x, k_y$  are the horizontal wave number  $k^2 = k_x^2 + k_y^2$  and  $n$  is the growth rate of harmonic disturbance

$$\frac{\rho}{\varepsilon} n u + i(U_x k_x + U_y k_y) u = [-ik_x \delta p + H_y (-ik_x h_y + ik_y h_x)] + \frac{1}{\varepsilon} [\mu(D^2 - k^2)u] - \frac{\mu}{\lambda} u \quad \dots(7)$$

$$\frac{\rho}{\varepsilon} n v + i(U_x k_x + U_y k_y) v = [-ik_y \delta p + H_x (ik_x h_y - ik_y h_x)] + \frac{1}{\varepsilon} [\mu(D^2 - k^2)v] - \frac{\mu}{\lambda} v \quad \dots(8)$$

$$\begin{aligned} \frac{\rho}{\varepsilon} n w + i(U_x k_x + U_y k_y) w = & -D \delta p - g \delta \rho + H_y (ik_x h_z - h_y D) - H_x (h_x D - ik_x h_z) \\ & - \frac{k^2 T}{n \varepsilon} \delta(z - z_s) w + \frac{1}{\varepsilon} [\mu(D^2 - k^2)w] - \frac{\mu}{\lambda} w \end{aligned} \quad \dots(9)$$

$$n h_x = i(U_x k_x + U_y k_y) h_x - H u D \quad \dots(10)$$

$$nh_y = i(U_x k_x + U_y k_y)h_x - H v D \quad \dots (11)$$

$$nh_z = i(U_x k_x + U_y k_y)h_x - H w D \quad \dots(12)$$

$$n \delta \rho = -w D \rho \quad \dots (13)$$

$$ik_x u + ik_y v + Dw = 0 \quad \dots(14)$$

$$ik_x h_x + ik_y h_y + Dh_z = 0 \quad \dots (15)$$

Eliminating some of the variables from the above equation, we obtain an equation in  $w$  as given below.

$$\begin{aligned} \frac{n'}{\varepsilon} [-D(\rho Dw) + \rho k^2 w] - \frac{gk^2}{n'^2} (D\rho) w + \frac{1}{n'^2} (H)^2 (D^2 - K^2) D^2 w + \frac{\mu}{\varepsilon} (D^2 - k^2)^2 w \\ + \frac{1}{\lambda} [\mu w k^2 - D(\mu Dw)] - \frac{k^4 T}{n} \delta(z - z_s) w = 0 \end{aligned} \quad \dots(16)$$

$$\text{Where } D = \frac{d}{dz} \quad \text{and} \quad n + i(k \cdot U) = n'$$

### 3. Superposed fluids

Consider the case in which two superposed fluids, occupying the regions  $z > 0$  and  $z < 0$ , are separated by a horizontal boundary at  $z = 0$ . In the two regions of constant density  $\rho$ , equation (16) becomes

$$(D^2 - k^2)(D^2 - M^2)w = 0 \quad \dots(17)$$

$$\text{Where } M^2 = \frac{\frac{\mu k^2}{\varepsilon} - \frac{\mu}{\lambda} + \frac{n'\rho}{\varepsilon}}{\frac{H^2}{n'\varepsilon} + \frac{\mu}{\varepsilon}} \quad \dots(18)$$

We assumed that the fluid of density  $\rho_1$ , viscosity  $\mu_1$ , and streaming velocities  $U_1 = (U_{x1}, U_{y1}, 0)$  occupy the lower region  $z < 0$ , while the fluid of density  $\rho_2$ , viscosity  $\mu_2$ , and streaming velocities  $U_2 = (U_{x2}, U_{y2}, 0)$  occupy the upper region  $z > 0$ . We then sought the solution of Eq. (8) for the 2 fluids moving with the presence of magnetic field  $H$  and flowing through a porous medium of porosity  $\varepsilon$ .

Since  $w$  must be bounded both when  $z \rightarrow +\infty$ , in the upper fluid, and  $z \rightarrow -\infty$ , in the lower fluid, the solutions of Eq. (8), which remained bounded in the two regions, are:

$$w_1 = P_1 n'_1 e^{kz} + Q_1 n'_1 e^{M_1 z} \quad (z < 0) \quad \dots(19)$$

$$w_2 = P_2 n'_2 e^{-kz} + Q_2 n'_2 e^{-M_2 z} \quad (z > 0) \quad \dots(20)$$

Where  $P_1, P_2, Q_1, Q_2$  are constants and  $M_1$  and  $M_2$  are the positive square roots of equation (18) for the two regions.

The expression for  $M_1^2$  and  $M_2^2$  are so defined that their real parts are positive i.e.

$$M_1^2 = k^2 \left[ 1 - \frac{\varepsilon}{\lambda k^2} + \frac{n'\rho_1}{k^2 \mu_1} - \frac{H_1^2}{n'\mu_1} + \frac{\varepsilon H_1^2}{n'\lambda k^2 \mu_1} \right] \quad \dots(21)$$

$$M_2^2 = k^2 \left[ 1 - \frac{\varepsilon}{\lambda k^2} + \frac{n'\rho_2}{k^2 \mu_2} - \frac{H_2^2}{n'\mu_2} + \frac{\varepsilon H_2^2}{n'\lambda k^2 \mu_2} \right] \quad \dots(22)$$

### 4. Boundary Conditions

For determining the four constant  $P_1, P_2, Q_1, Q_2$  we require the four boundary conditions.

$$\text{Three conditions require continuity of } w, Dw, \text{ and } \mu(D^2 + k^2)w \quad \dots(23a,b,c)$$

at the interface  $z = 0$

We obtain fourth condition by integrating equation (21)

$$\left[ \left\{ \rho_2 - \frac{H_2^2}{n'^2_2} (D^2 - k^2) - \frac{\mu_2}{n'_2} (D^2 - k^2) + \frac{\mu_2 \varepsilon}{\lambda n'_2} \right\} Dw_2 \right]_{z=0} - \left[ \left\{ \rho_1 - \frac{H_1^2}{n'^2_1} (D^2 - k^2) - \frac{\mu_1}{n'_1} (D^2 - k^2) + \frac{\mu_1 \varepsilon}{\lambda n'_1} \right\} Dw_1 \right]_{z=0} + \frac{k^4}{n^2} T w_0 + gk^2 \left( \frac{\rho_2}{n'^2_2} - \frac{\rho_1}{n'^2_1} \right) w_0 + 2k^2 \left( \frac{\mu_2}{n'_2} - \frac{\mu_1}{n'_1} \right) Dw_0 = 0 \quad \dots(24)$$

Where the subscripts 1 and 2 denote the corresponding quantities at the lower and upper fluids respectively and  $w_0$  and  $(Dw)_0$  are the unique values of these quantities at  $z = 0$

## 5. Dispersion relations

After applying the boundary conditions (23a, b, c) and (24) to equation (19) and (20) we get the dispersion relations as

$$P_1 + Q_1 = P_2 + Q_2 \quad \dots (25)$$

$$kP_1 + M_1 Q_1 = -kP_2 - M_2 Q_2 \quad \dots (26)$$

$$\mu_1 [2k^2 P_1 + (M_1^2 + k^2) Q_1] = \mu_2 [2k^2 P_2 + (M_2^2 + k^2) Q_2] \quad \dots (27)$$

$$\rho_2 (-kP_2 - M_2 Q_2) - \rho_1 (kP_1 + M_1 Q_1) + \left\{ \frac{\varepsilon \mu_2}{\lambda n'_2} (-kP_2 - M_2 Q_2) + \frac{\varepsilon \mu_1}{\lambda n'_1} (kP_1 + M_1 Q_1) \right\} + \frac{H_2^2}{n'^2_2} (M_2^2 - k^2) M_2 Q_2 + \frac{H_1^2}{n'^2_1} (M_1^2 - k^2) M_1 Q_1 + \frac{\mu_2}{n'_2} (M_2^2 - k^2) M_2 Q_2 + \frac{\mu_1}{n'_1} (M_1^2 - k^2) M_1 Q_1 = -\frac{gk^2}{2} \left( \frac{\rho_2}{n'^2_2} - \frac{\rho_1}{n'^2_1} \right) (P_1 + Q_1 + P_2 + Q_2) - \frac{k^4 T}{n} w_0 (P_1 + Q_1 + P_2 + Q_2) - k^2 \left( \frac{\mu_2}{n'_2} - \frac{\mu_1}{n'_1} \right) (kP_1 + M_1 Q_1 - kP_2 - M_2 Q_2) \quad \dots(28)$$

Eliminating  $P_1, P_2, Q_1$ , and  $Q_2$  from equations (25), (26), (27), (28), we get

$$\begin{vmatrix} 1 & 1 & -1 & -1 \\ k & M_1 & k & M_2 \\ 2k^2 \alpha_1 v_1 & \alpha_1 v_1 (M_1^2 + k^2) & -2k^2 \alpha_2 v_2 & -\alpha_2 v_2 (M_2^2 + k^2) \\ -\alpha_1 k + \frac{kR}{2} - \frac{\varepsilon k \alpha_1 v_1}{\lambda n'_1} + Ck & -\frac{\varepsilon \alpha_1 v_1 M_1}{\lambda n'_1} - \alpha_1 M_1 + \frac{v_1^2 (M_1^2 - k^2) M_1}{n'^2_1} + \frac{kR}{2} + \frac{\alpha_1 v_1 (M_1^2 - k^2) M_1}{n'_1} + CM_1 & -\alpha_2 k + \frac{kR}{2} - \frac{\varepsilon k \alpha_2 v_2}{\lambda n'_2} + Ck & -\frac{\varepsilon \alpha_2 v_2 M_2}{\lambda n'_2} - \alpha_2 M_2 + \frac{v_2^2 (M_2^2 - k^2) M_2}{n'^2_2} + \frac{kR}{2} + \frac{\alpha_2 v_2 (M_2^2 - k^2) M_2}{n'_2} - CM_2 \end{vmatrix} = 0 \quad \dots (29)$$

$$(M_1 - k) [2k^2 (\alpha_1 v_1 - \alpha_2 v_2) \{ (\alpha_2 + C) (M_2$$

$$-k) - \frac{\alpha_2 v_2 \varepsilon}{\lambda n'_2} (M_2 - k) + \frac{v_2^2}{n'^2_2} (M_2^2 - k^2) M_2 + \frac{\alpha_2}{n'_2} v_2 (M_2^2 - k^2) M_2 \}$$

$$+ v_2 \alpha_2 (M_2^2 - k^2) \{ k(R - 1) + \frac{k\varepsilon}{\lambda} \left( \frac{\alpha_1 v_1}{n'_1} + \frac{\alpha_2 v_2}{n'_2} \right) \}] - 2k [v_2 \alpha_2 ($$

$$M_2^2 - k^2) \{ (-\alpha_1 + C) (M_1 - k) + \frac{\alpha_1 v_1 \varepsilon}{\lambda n'_1} (M_1 + k) - \frac{v_1^2}{n'^2_1} (M_1^2 - k^2) M_1 + \frac{\alpha_1}{n'_1} v_1 (M_1^2 - k^2) M_1 \} +$$

$$v_1 \alpha_1 (M_1^2 - k^2) \{ -(\alpha_2 + C) (M_2 - k) - \frac{\alpha_2 v_2 \varepsilon}{\lambda n'_2} (M_2 - k) + \frac{v_2^2}{n'^2_2} (M_2^2 - k^2) M_2 + \frac{\alpha_2}{n'_2} v_2 (M_2^2 - k^2) M_2 \}]$$



$$\begin{aligned}
& + (M_2 - k) \left[ v_1 \alpha_1 (M_1^2 - k^2) \left\{ k(R - 1) + \frac{k\varepsilon}{\lambda} \left( \frac{\alpha_1 v_1}{n'_1} + \frac{\alpha_2 v_2}{n'_2} \right) - 2k^2 (\alpha_1 v_1 - \alpha_2 v_2) \right. \right. \\
& \left. \left. \{ (-\alpha_1 + C) (M_1 - k) + \frac{\alpha_1 v_1 \varepsilon}{\lambda n'_1} (M_1 + k) - \frac{V_1^2}{n'^2_1} (M_1^2 - k^2) M_1 + \frac{\alpha_1}{n'_1} v_1 (M_1^2 - k^2) M_1 \} \right] = 0 \right. \\
& \left. \dots (30) \right.
\end{aligned}$$

Where

$$\begin{aligned}
R &= g k \left( \frac{\alpha_2}{n'_2} - \frac{\alpha_1}{n'_1} \right), \quad C = k^2 \left( \frac{\alpha_2 v_2}{n'_2} - \frac{\alpha_1 v_1}{n'_1} \right), \quad V_{1,2}^2 = \frac{H_{1,2}^2}{\rho_{1,2}}, \quad v_{1,2} = \frac{\mu_{1,2}}{\rho_{1,2}}, \quad \alpha_{1,2} = \frac{\rho_{1,2}}{\rho_1 + \rho_2}, \\
& (\alpha_1 + \alpha_2 = 1)
\end{aligned}$$

The values of  $M_1$  and  $M_2$  are given by the square roots of Eqs. (21) and (22). To obtain the values of  $M_1$  and  $M_2$ , we used the binomial theorem and retained terms up to  $\frac{1}{v_{1,2}}$  as in the case of nonstreaming fluids (Bhatia 1974). We could then write  $M_1$  and  $M_2$  as:

$$\begin{aligned}
M_1 &= k \left[ 1 - \frac{\varepsilon}{2\lambda k^2} + \frac{n'\rho_1}{2\mu_1 k^2} - \frac{H_1^2}{2n'\mu_1} + \frac{\varepsilon H_1^2}{2n'\lambda\mu_1 k^2} \right] \\
M_2 &= k \left[ 1 - \frac{\varepsilon}{2\lambda k^2} + \frac{n'\rho_2}{2\mu_2 k^2} - \frac{H_2^2}{2n'\mu_2} + \frac{\varepsilon H_2^2}{2n'\lambda\mu_2 k^2} \right]
\end{aligned}$$

On substituting the expression for  $M_1$  and  $M_2$ , taking  $v = v_1 = v_2$ ,  $V_1 = V_2 = V$  and  $U_1 = U$ ,  $U_2 = -U$  in Eq. (30), we obtained the characteristic equation. It is obvious that the expansion resorted to here is due to reasons of mathematical tractability. This enabled us to analyze the stability of the system. Substituting the values of  $M_1$  and  $M_2$ , given by Eqs. (21) and (22), in Eq. (20), we obtained the dispersion relation as

$$\sum_{i=0}^9 A_i n^i = 0 \quad \dots (31)$$

where the coefficients  $A_i$  ( $i = 1-10$ ) are complicated expressions involving the wave number  $k$  and the parameters  $\alpha_1$ ,  $\alpha_2$ ,  $U$ ,  $V$ ,  $v$ ,  $\varepsilon$ , and  $\lambda$  characterizing respectively, the effects of density, streaming velocity, the Alfvén Velocity, the viscosity, the porosity, and the permeability of the porous medium of the fluids. These coefficients are not given here as they are very lengthy.

## Conclusion

The dispersion relation given by Eq. (23) is quite complicated, particularly as the coefficient of  $A_i$  involve several parameters. It is thus not feasible to analyze the dispersion relation analytically. We therefore solved it numerically, for different values of the parameters, for an unstable arrangement of superposed fluids, i.e. a top-heavy configuration and the same.

We were interested in the qualitative behavior of the various parameters on the instability of the configuration. Therefore, the dispersion relation, Eq. (31), was numerically solved to ascertain the values of the growth rate against the wave number for various values of one parameter, taking fixed values of other parameters. The dispersion relation was first non dimensionalized by measuring  $n$  and the parameters in terms of  $\sqrt{g}$ . For an unstable system, we must have  $\alpha_1 < \alpha_2$  ( $\alpha_1 + \alpha_2 = 1$ ). The numerical calculations are presented in fig. 1-5.

The effect of kinematic viscosity on the system's stability is presented in Figures 1, in which we again plotted growth rate  $n$  versus wave number  $k$  for varying values of viscosity ( $v$ ). From Figures 1, we notice that the kinematic viscosity had a stabilizing influence on the instability of the system, as the increase in viscosity tended to decrease the value of  $n$  for the same  $k$ . Several other authors have examined the effect of viscosity on the stability of different hydrodynamic and hydromagnetic systems. For nonstreaming superposed fluids, Bhatia (1974), and Kumar et al. (2007) have pointed out the stabilizing character of viscosity on the stability of the system. The

result obtained in the present paper is thus in agreement with the observations of earlier researchers. The influence of surface tension on the system’s stability is presented in Figures 2, in which we again plotted growth rate  $n$  versus wave number  $k$  for varying values of surface tension ( $T$ ). From Figures 2, we notice that the surface tension had a stabilizing influence on the instability of the system, as the increase in surface tension tended to decrease the value of  $n$  for the same  $k$ .

The effect of streaming velocity on the system’s stability is presented in Figures 3 , in which growth rate  $n$  is plotted versus wave number  $k$  for varying values of streaming velocity and We found that the more the values of streaming velocity increased for a fixed wave number, the larger the values of the growth rate were. The streaming velocity thus had a destabilizing influence on the system. The effect of streaming motion on the stability of a superposed fluid has been investigated by several researchers in the past, such as Watson et al. (2004), and Bhatia and Mathur (2006). For nonporous fluid media, they all found that the streaming motion had a destabilizing influence on the system. The result obtained in the present paper for the effect of streaming motion in porous fluids thus agrees with earlier findings.

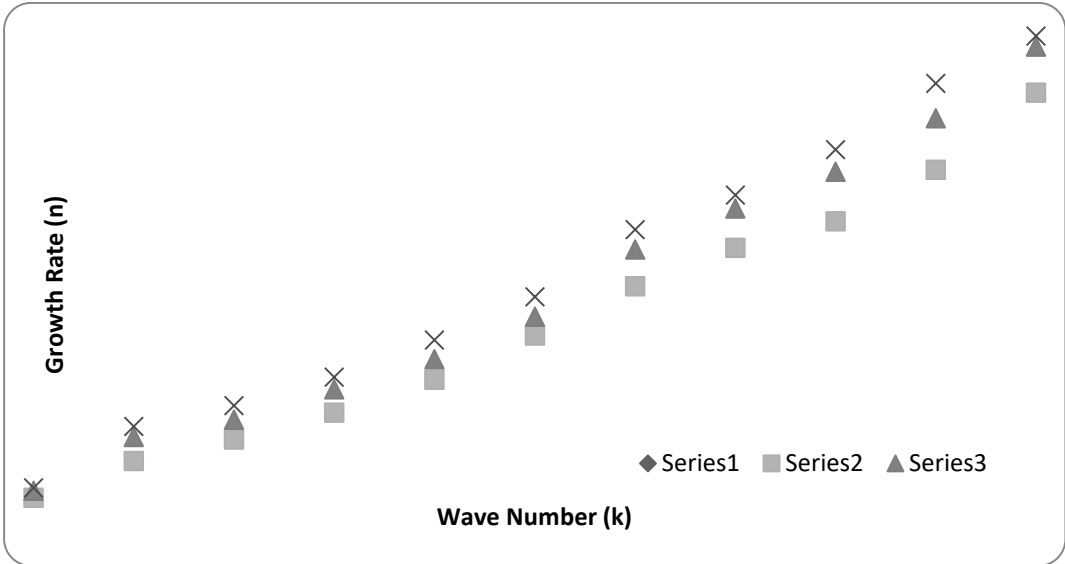


Fig. 1 Dependence of growth rate (real positive  $n$ ) against wave number  $k$  for kinematic viscosity  $v= 3, 4, 5$  when  $\alpha_1= 0.25, \alpha_2=0.75, V = 1, U =1, \lambda = 1, \varepsilon = 0.1$ ,

--- ▲ --for $v =3$	--- ■ -- for $v =4$	--◆--for $v = 5$
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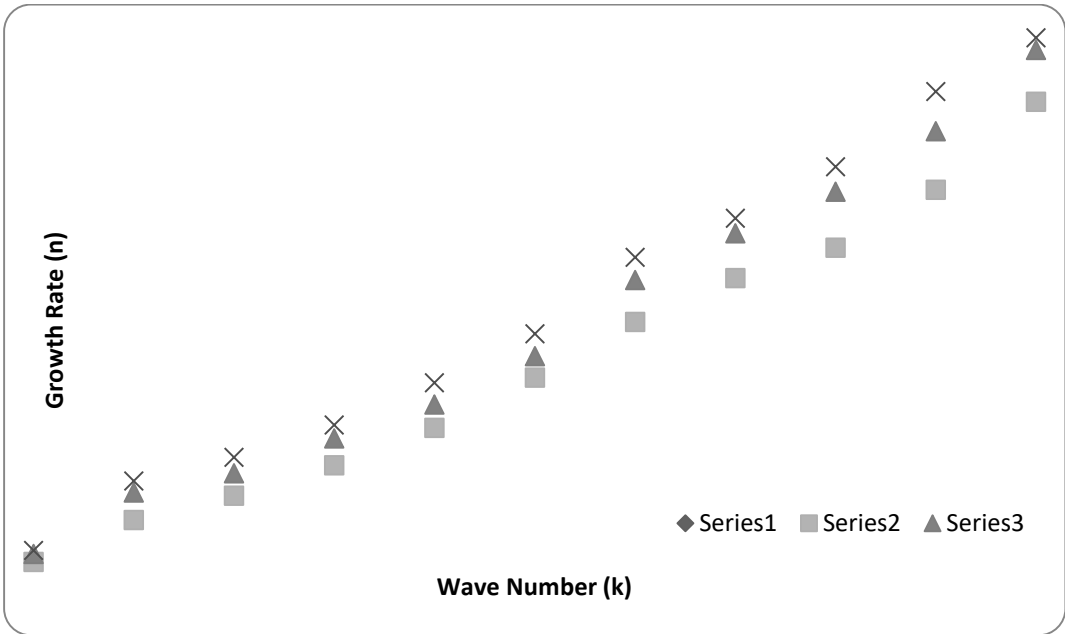


Fig. 2 Dependence of growth rate (real positive  $n$ ) against wave number  $k$  for surface tension  $T = 1,2,3$  when  $\alpha_1= 0.25, \alpha_2=0.75, V = 1, v = 1, U =1, \varepsilon = 0.1$

---x-for $T =1$	--- ▲ — for $T =2$	--- ■ -- for $T = 3$
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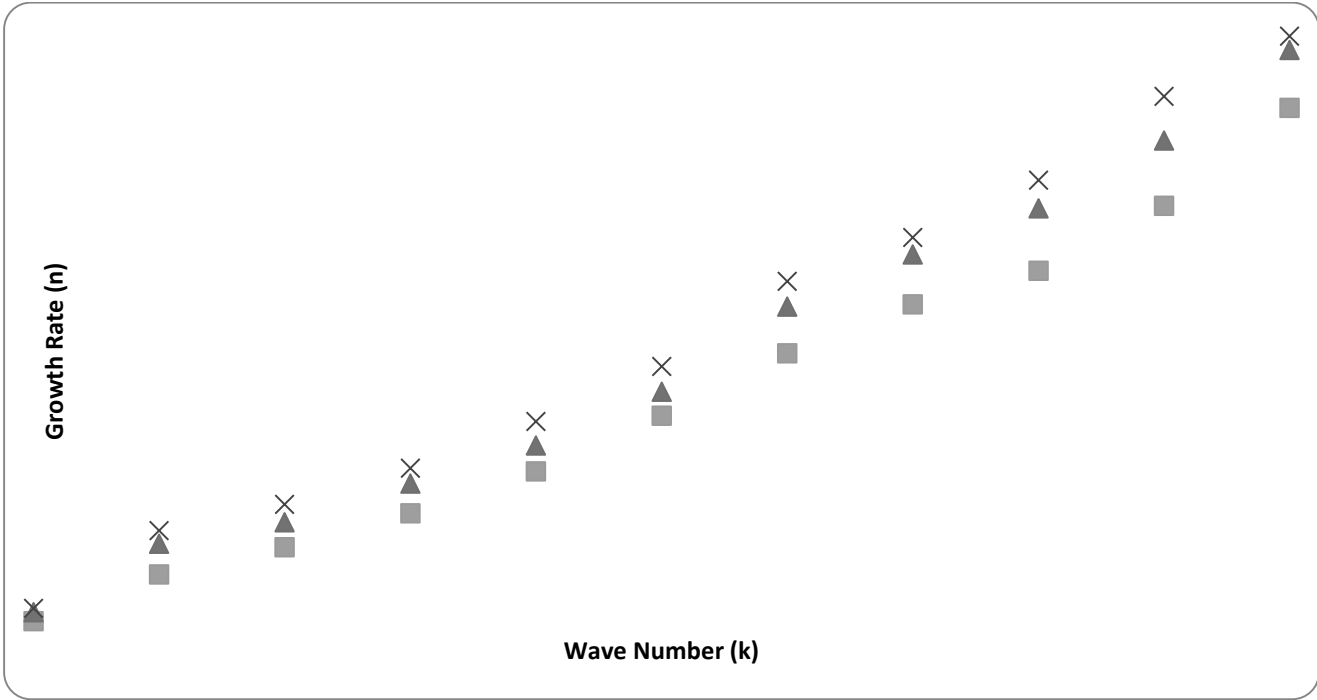


Fig. 3 Dependence of growth rate (real positive  $n$ ) against wave number  $k$  for stream velocity  $U = 1, 2, 3$  when  $\alpha_1 = 0.25$ ,  $\alpha_2 = 0.75$ ,  $V = 1$ ,  $\nu = 1$ ,  $\lambda = 1$ ,  $\varepsilon = 0.1$

--◆--for $U = 1$	--- ■ --- for $U = 2$	--- ▲ --- for $U = 3$
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Thus we conclude that kinematics viscosity and surface tension have stabilizing influence and the streaming velocity has destabilizing influence on the system.

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