



Laser Second Harmonic Generation in Carbon Nanotubes

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Abstract

In this current study, we have investigated the phenomenon of second harmonic generation through the nonlinear interaction of Laguerre-Gaussian lasers with arrays of vertically aligned carbon nanotubes. These carbon nanotube arrays are structured on a dielectric surface, and the Laguerre-Gaussian laser beam interacts perpendicularly to the length of the carbon nanotubes. As the field of the Laguerre-Gaussian laser beam engages with the carbon nanotube arrays, the nonlinearity within the medium is induced by the electrostatic restoration force, leading to the displacement of the electron cylinder in relation to the ion cylinder. This nonlinearity gives rise to second and third harmonic electron excursions within the carbon nanotubes. We have derived analytical expressions for the second harmonic nonlinear current density and the second harmonic electric field in carbon nanotubes. The findings and discussions of this theory are illustrated through various graphical representations. Due to the presence of surface plasmon resonance, an enhanced amplitude of the second harmonic electric field is observed when the laser beam frequency approaches approximately $\sqrt{2}$ times the electron plasma frequency. The Laguerre-Gaussian laser beam generates a significantly larger amplitude of the second harmonic compared to the Gaussian laser beam alone.

Keywords: Second harmonic generation, Current density, laser beam width, beam index, collisional frequency, carbon nanotube radii, Carbon nanotube, Restoration force

1. Introduction

In last few years, the nonlinear interactions of different profile laser beam with plasma embedded nano dimension medium has revealed variety of nonlinear phenomenon such as harmonic generation, self-focusing, soft x-ray generation, plasma diagnostic [1], anomalous absorption, electron heating, terahertz radiation generation [2], electrostatic wave excitation, charged particle acceleration, parametric instability [3]. Due to possess of highly carrier (electron and ion) mobility, highly mechanical tensile strength, charge storage capacity and

vertical aligned property, it promises several technological applications [4-5]. Presence of effective plasmons resonance on the surface of conducting materials and nanoparticles promises the anomalous nonlinear property. In spherical nanoparticles, the effective surface plasmons resonance phenomenon is seen as the laser beam frequency approaches $\sqrt{3}$ times of electron plasma frequency inside the spherical nanoparticles while in cylindrical shape nanoparticles, the effective surface plasmons resonance phenomenon is seen as the laser beam frequency approaches near the $\sqrt{2}$ times of electron plasma frequency inside the cylindrical nanoparticles [6-7]. This depicts that resonant condition is observed earlier in the case of carbon nanotube as compared to spherical nanoparticles. The size of nanoparticles and inter particle separation can change the resonance position with without variation of laser beam frequency.

In the present theoretical investigation, we study a scheme of second harmonic generation Laguerre-Gaussian laser beam in vertical aligned carbon nanotube arrays. As the high power Laguerre-Gaussian laser beam interacts with carbon nanotube, then atoms of this medium are get ionized and eventually become the plasma balls. By this, the generation of preformed plasma is taken into the place. The orbital angular momentum associated with this laser beam can be enhanced the amplitude of second harmonic electric field. The pictorial schematic of this laser beam second harmonic generation in carbon nanotube is shown in Fig. 1. The nonlinear interaction of laser beam with carbon nanotube is given in Sec. 2. In Sec. 3, we derive the analytical expression of second harmonic electric field amplitude. In Sec. 4, the results and discussion of this proposed theory is explained. Finally, the summary and conclusions of this theory is presented in Sec. 5.

2. Nonlinear Interaction

Consider a vertically aligned carbon nanotube arrays are rested in a dielectric surface (say Glass). The microwave plasma-enhanced chemical vapour deposition synthesized technique can be engineered to obtain the deformed vertically aligned carbon nanotubes (VA-CNTs). These CNTs obtained by this proposed method are very soften and more prove to ripples as compared to crystalline CNTs. The assembly comprises N nanotubes per unit area, placed parallel to each other with axis along \hat{z} . the CNTs have interior free electron density n_{e0} , radius r_{c0} , length L and the separation between the CNTs axes is d.

Herein, we have taken a high power Laguerre-Gaussian (LG) laser beam with cylindrical symmetric intensity distribution that propagate coaxially along the z-direction in VA-CNTs (vertically aligned in arrays of carbon nanotubes). It is supposed that laser field amplitude is polarized along r-direction. The Laguerre-Gaussian laser beam can be described by an axial phase singularly with radial index p and the strength l, which is called optical vortex change

number (or azimuthal mode index). The general algebraic form of Laguerre-Gaussian laser beam electric field profile can be expressed as

$$\vec{E}_L(r, \theta, z) = \hat{r} E_{0L} \left(\frac{\sqrt{2}r}{w_{0L}} \right)^l e^{\left(-\frac{r^2}{w_{0L}^2} - il\theta \right)} L_p^l \left(\frac{2r^2}{w_{0L}^2} \right) e^{-i(\omega t - kz)}, \quad (1)$$

where r is the radial distance from the center, θ is the Azimuthal angle, w_{0L} represents LG laser beam width, L_p^l denotes the generalized Laguerre polynomial, l and p is the azimuthal index and radial index associated with Laguerre polynomial and E_{0L} is the initial electric field amplitude of Laguerre-Gaussian laser beam.

The atoms of carbon nanotubes are eventually ionized by the interaction of Laguerre-Gaussian laser beam. This results the formation of preformed plasma in carbon nanotube. Since, we know that ions possess heavier mass as compared with electron. Hence, electron cylinder of carbon nanotube atoms is displaced with respect to ion cylinder (in rest) and we can represent this excursion as $\vec{\Delta}$. The electron excursion is zero along the axis of vertically aligned carbon nanotube and non-zero along the laser field polarization direction $r(x, y)$. A self-consisted space charge field is generated in between the overlapping region of electron and ion-cylinder owing to excursion of electron in carbon nanotube. The formula of self-consisted space charge field is defined by $n_{co} e \vec{\Delta} / 2\epsilon$ in this overlapped region where $\epsilon = \epsilon_r \epsilon_0$ is the electrical permittivity of medium and is the dielectric constant (relative permittivity) of medium. The self-generated electric field will be the sum of electric field produced by electron cylinder and ion cylinder. At point $(r' \varphi z)$, the interior electric field field on eletron cylinder produced by the interaction of Laguerre-Gaussian laser beam can be written as

$$\vec{E}_e = \frac{n_{ceo} e (\vec{r}' - \vec{\Delta})}{2\epsilon_r \epsilon_0}, \quad (2)$$

At point $(r' \varphi z)$, the interior electric field on ion cylinder produced by Laguerre-Gaussian laser beam can be written as

$$\vec{E}_i = \frac{n_{ceo} e r_{co}^2 \vec{r}'}{2\epsilon_r \epsilon_0 r'^2}, \quad (3)$$

3. Second Harmonic Generation

If we consider that the interaction length is greater as compared with the inverse of wave number mismatch $(k_2 - 2k_1)^{-1}$, then the operator ∇ can be replaced by $\nabla \rightarrow i2k_1$. In this consideration, we can solve the Eq. (18) as follows

$$\vec{E}_2 = \frac{\omega^2}{c^2} \times n_{ce0} e \times \frac{1}{(k_2^2 - 4k_1^2)} \times \frac{r_{c0}^2}{d^2} \times \frac{\Delta_1^2}{\left(4\omega^2 - \frac{\omega_{pe}^2}{2} + i2\nu_{ei}\omega\right)}, \quad (4)$$

$$\text{where } k_1^2 = \frac{4\omega^2}{c^2} \left(1 - \frac{\pi r_{c0}^2}{d^2} \frac{\omega_{pe}^2}{\left(\omega^2 - \frac{\omega_{pe}^2}{2} + i\nu_{ei}\omega\right)} \right).$$

4. Results and Discussion

Laguerre-Gaussian laser beam second harmonic generation is studied in vertically aligned arrays of carbon nanotube. Laguerre-Gaussian laser beam is particular type of laser beam having cylindrical symmetric field distribution. As this laser beam interacts with carbon nanotube medium, the atoms of this nano dimension medium quickly ionized and eventually converted into the plasma balls. The nonlinear interaction of Laguerre-Gaussian laser beam with carbon nanotube promises the enhanced second harmonic generation. Owing to possess of maximum nonlinearity, we have taken the electron excursion as in the direction along the laser beam polarization. The possible schematic of laser second harmonic generation in carbon nanotube is given in Fig. 1. The effect of laser beam frequency, laser beam width, electron collisional frequency, orbital angular momentum parameter (l), radial index (p), density of carbon nanotube medium, carbon nanotube radii, inter carbon nanotube separation and laser beam transverse propagation distance on the generated second harmonic nonlinear current density and second harmonic electric field amplitude have been graphically discussed.

Fig. 2 shows the variation of Laguerre-Gaussian laser beam normalized intensity distribution as a function of laser beam transverse propagation distance for different values of laser beam indices l, p. The intensity distribution is attained purely Gaussian profile for indices l=p=0. The peak of intensity profile is appeared at the origin of laser beam propagation. As one increases the Laguerre-Gaussian indices l=p=1, the intensity profile attains two lobes having doughnut shape. The first one peak is appeared at laser transverse propagation radial distance $r/w_{0L} \sim 0.48$ and second one peak is appeared at laser beam transverse radial distance $r/w_{0L} \sim 1.50$. It is to be noticed that the amplitude of first one peak is greater as compared with second one peak. Further, if we increase the Laguerre-Gaussian laser beam indices l=p=2, the intensity profile attains three lobes with doughnut shaped. The first peak is appeared at $r/w_{0L} \sim 0.58$, the second one peak is appeared at $r/w_{0L} \sim 1.34$ and the third one peak is appeared at $r/w_{0L} \sim 2.21$. The amplitude of first one peak is much stronger than both two other peaks. The amplitude of second one peak is slightly grater than third one peak. The appearance of multiple lobes promises the much more nonlinear interaction as compared with single lobe. The Physics behind this laser beam intensity distribution can be explained via laser beam

indices l, p . The presence of orbital angular momentum (azimuthal index) l , can enhance the transfer of energy and momentum to the laser beam. Similarly, the radial index p , enhance the phase front of laser beam radial propagation. In this way, we can say that these indices l, p enhance the lobes and amplitude of laser beam intensity distribution.

The variation of first order normalized electron excursion as a function of normalized laser beam frequency for different values of Laguerre-Gaussian laser beam indices l, p . The peak of electron excursion is appeared at normalized laser beam frequency $\omega/\omega_{pe} \sim 0.707$. This Laguerre-Gaussian laser beam frequency is typically equal to the $\sqrt{2}$ times the electron plasma frequency and which corresponds to effective surface plasmons frequency on the surface of vertically aligned nanotube. Since, at this particular value of laser beam frequency, laser beam efficiently absorbs on the surface of carbon nanotubes. This resonant laser beam frequency causes to impartation of large velocity to the electron. Hence, efficient large electron excursion is attained by resonant laser beam frequency. Further, one can see that with increase in Laguerre-Gaussian laser beam indices (l, p), the amplitude of electron excursion is enhanced. The increased laser beam indices cause the transfer of large amount energy-momentum to the surface of carbon nanotube and by the result electron associated with its might be energised. Therefore, as one takes

5. Summary and Conclusions

In this present theoretical investigation, second harmonic generation is studied by the interaction of high power Laguerre-Gaussian laser beam in vertically aligned carbon nanotube arrays. By using the dynamics of nonlinear properties of electron in carbon nanotube, the analytical expressions of electron first harmonic, second harmonic, third harmonic oscillatory velocity, restoration force, second harmonic nonlinear current density and second harmonic electric field amplitude are derived. The various graphical results promise for enhanced laser second harmonic generation in carbon nanotube with the variation of laser beam transverse propagation distance, laser beam width, Laguerre-Gaussian beam index, beam frequency, carbon nanotube radii, inter carbon nanotube separation, electron-ion collisional frequency and equilibrium electron density of carbon nanotube. The presence of effective surface plasmons oscillation on the surface of carbon nanotube might be caused the resonant generation of second harmonic electric field. It was observed that as the laser beam propagation distance become typically 0.46 times of laser beam width, one can achieve the large electric field amplitude of second harmonic. The laser second harmonic electric field amplitude can be tuned and controlled by the various laser and carbon nanotube parameters.

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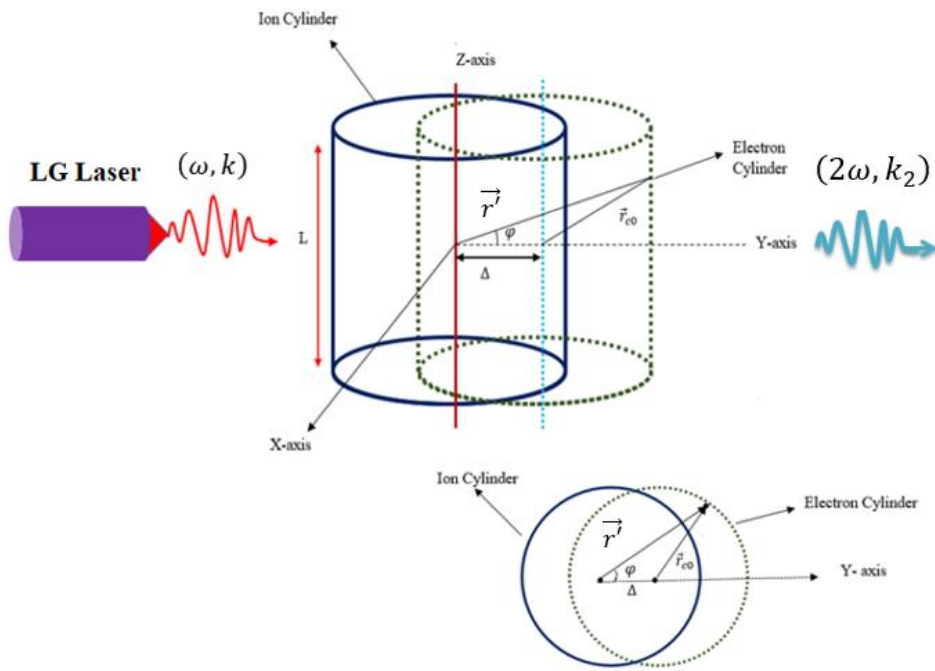


Fig. 1

Fig. 1: Schematic diagram of laser second harmonic generation in VA carbon nanotube

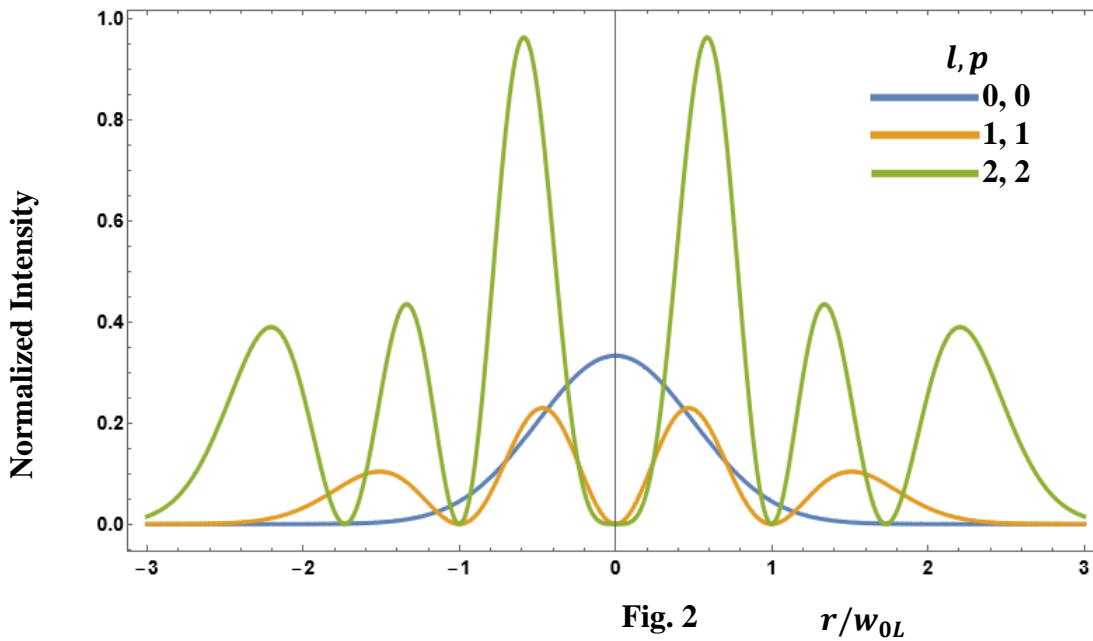


Fig. 2

Fig. 2: Variation of normalized Laguerre-Gaussian laser beam intensity profile with normalized laser beam propagation distance for different values laser mode index $l, p = (0, 0, 1, 1, 2, 2)$

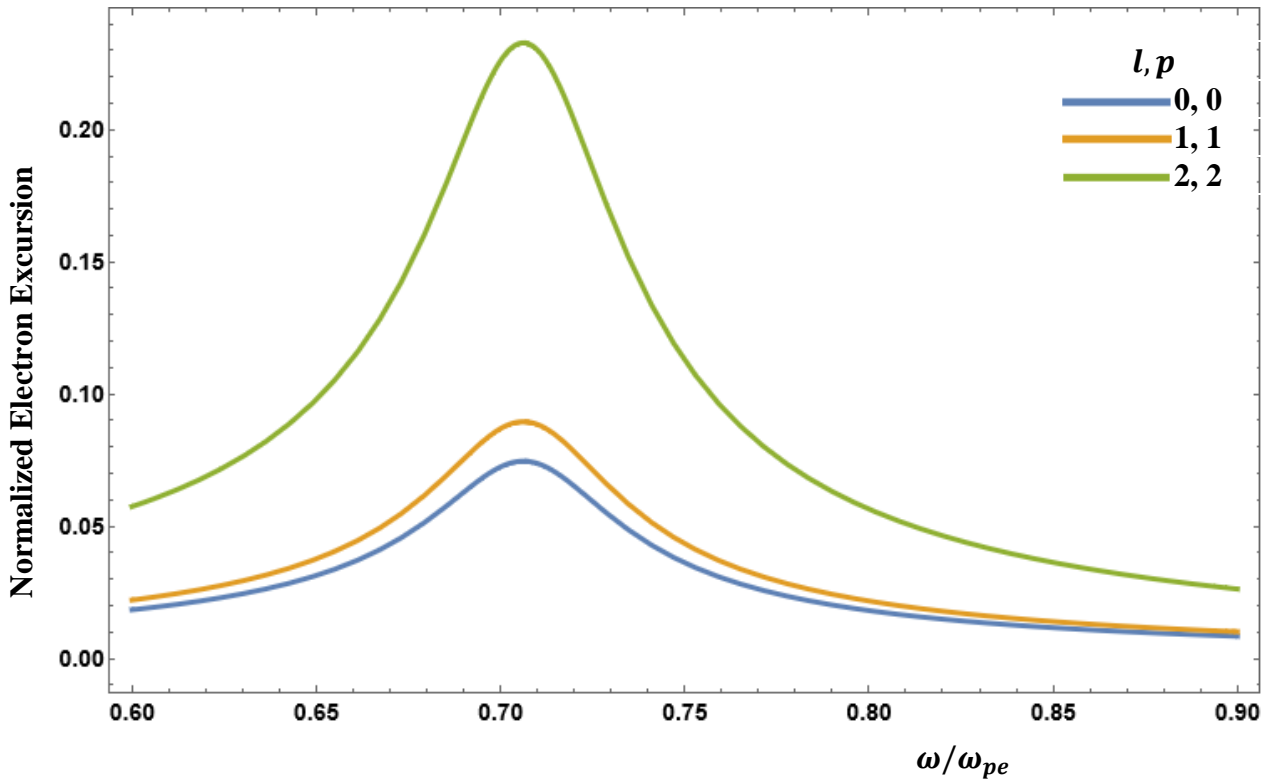


Fig. 3

Fig. 3: Variation of normalized electron excursion with normalized laser beam frequency for different values of laser mode index $l, p = (0, 0), (1, 1), (2, 2)$ when $n_{ce0}/n_0 = 0.01$, $\xi = eE_{0L}/m\omega_{pe}^2 r_{c0} = 0.1$, $r_{c0} = 20 \text{ nm}$, $r_{c0}/b = 0.1$, $v_{ei}/\omega_{pe} = 0.01$, $r/w_{0L} = 0.5$