Concept of Rational Number Series

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Abstract

A rational number series is a natural way to express numbers. Numbers over ages has been expressed in many forms of series including triangle series, Fibonacci series, e series, trigonometric series and others. In this paper there is an outline of the rational number series. The rational series expressions result in geometric progression till infinity; and some geometric progression series till finite terms. Also rational series expressions can be considered in using as a prime number generator.

Introduction

Humans have associated numbers with identifying similarity in objects and the early man started counting 1 tree, 2 trees, 3 trees and so on. The early man realized that there is a gap between two trees which made them distinct. Also the early man realized that the two distinct objects had some commonality which made him say tree and he counted as 1 tree, 2 trees and so on. Much later the concept of the mathematical constant e was given by Leonhard Euler. The era of mathematics had begun and much -much complex aspects of the physical world and the machine world have the usage of e in them.

Another aspect associated with numbers is growth of a living object. Like 1 foot high tree, 2 feet high tree, 3 feet high tree etc. The early man used the length of the hand, feet etc. to count the height. Trigonometry was developed by mathematicians to calculate the height of not only trees but also tall structures. But today trigonometry is a vast field of mathematics and the trigonometric series and expressions find usage in the modern world in practically all areas of calculations.

However, a living thing is associated with the presence of another thing. This is a reality. A plant is dependent on the existence of water. A plant needs manures and fertilizers. It is a

rational relationship and hence expressions like 2 mugs of water for 1 plant three handfuls of manure for 2 plants etc. do arise in our daily life. This stresses the importance of numbers like 1/2, 2/3 etc.

Also humans have a method of having a common name and an individual name. Like a bag of wheat or a bag of rice is considered a bag of cereal. So 1 bag of wheat among 2 bags of cereal can be represented as 1/2. 2 bags of rice among 3 bags of cereal is represented as 2/3.

There is an importance of rational numbers like 1/2, 2/3 in our daily lives. This aspect has not been considered in our number systems. Though these are nature blessed, rational expressions based on rational series does not exist.

I have outlined two expressions on the rational growth of this rational number systems. A rational series can be represented as a series 1/2, 2/3, 3/4 …. or as a series 1/2,3/4,5/6 . . . . . There are two expressions which I present in this paper which use the above mentioned two series. The results here are accurate.

I have also used the series $1^2/2^2$, $3^2/4^2$, $5^2/6^2$, $7^2/8^2$ . . . . . . as a prime number generator. The number of prime number generated by the function used is about 30%. However, if we eliminate certain obvious non-primes by checking with certain prime numbers like 5, 11 . . . . etc. then the results improve. And about 50-60% of the results are prime.

Also I have used the series $1^2/2^2$, $3^2/4^2$, $5^2/6^2$, $7^2/8^2$ . . , $1^2/2^2$, $2^2/3^2$, $3^2/4^2$, $4^2/5^2$ . . . . ., $\sqrt{1}/\sqrt{2}$, $\sqrt{3}/\sqrt{4}$, $\sqrt{5}/\sqrt{6}$, $\sqrt{7}/\sqrt{8}$ . . and $\sqrt{1}/\sqrt{2}$, $\sqrt{2}/\sqrt{3}$, $\sqrt{3}/\sqrt{4}$, $\sqrt{4}/\sqrt{5}$. . . . to list out results which are important. There are four results based on them.

IDENTIFICATIONS

Rational number series are 1/2, 3/4, 5/6, 7/8 . . . . and so on. (A)

Rational number series are also 1/2, 2/3, 3/4, 4/5, 5/6 . . . . and so on.(B)

Triangle number series are the series 1,3,6,10,15,21 . . . . and so on.(C)

Prime numbers are the numbers which do not have factors apart from itself and 1.

Examples are 2, 3, 5, 7, 11, 13 etc.(D)

Rational square series are $1^2/2^2$, $3^2/4^2$, $5^2/6^2$, $7^2/8^2$ . . . . and so on. (E)

Rational square series are also $1^2/2^2$, $2^2/3^2$, $3^2/4^2$, $4^2/5^2$ . . . . and so on.(F)

Rational square root series are $\sqrt{1}/\sqrt{2}$, $\sqrt{3}/\sqrt{4}$, $\sqrt{5}/\sqrt{6}$, $\sqrt{7}/\sqrt{8}$ . . . . . . and so on.(G)

Rational square root series are also $\sqrt{1}/\sqrt{2}$, $\sqrt{2}/\sqrt{3}$, $\sqrt{3}/\sqrt{4}$, $\sqrt{4}/\sqrt{5}$. . . . . . and so on.(H)

Results and Interpretations

Result 1

We have from (C), that a triangle series is 1, 3, 6, 10, 15, 21 . . . .

And from (A), 1/2, 3/4, 5/6, 7/8 . . . . and so on.

There is a relationship between the rational series (A) and the triangle series (C). And that is

3/4 - 1/2 = 1/(4x 1); where 1 is the first element of the tree series 1, 3, 6, 10, 15, 21 . .

5/6 - 3/4 = 1/(4x 3); where 3 is the second element of the tree series 1, 3, 6, 10, 15, 21 . . . . .

7/8 - 5/6 = 1/(4x 6); where 6 is the third element of the tree series 1, 3, 6, 10, 15, 21 . . . . .

9/10 - 7/8 = 1/(4x 10); where 10 is the fourth element of the tree series 1, 3, 6, 10, 15, 21 . . . .

. . . . . and so on.

Result 1 – Background

There is one classic puzzle which states "What are the weights needed to weigh any weight upto 100kgs ; when you are given a pair of scales?"

The trick here is that you can put 3 kgs on one side and 1 kgs on the other to make it possible that an object of 2 kgs is measured.

Over internet the best answer is claimed to be the 3^n series. The weights are 1, 3, 9, 27 and 81. 5 weights.

The second best answer (my answer inspired on triangle series) is 1,3,6,10,30,60. 6 weights.

The third best answer is the 2^n series with 1,2,4,8,16,32,64. 7 weights.

Definitely the 3^n series answer requires lesser number of weights. Only 5. But consider the amount of iron in kgs needed. 1+3+9+27+81 = 121 kgs of iron.

The second answer requires 1+3+6+10+30+60 = 110 kgs of iron.

The third answer requires 1+2+4+8+16+32+64 = 127 kgs of iron.

If the puzzle was re-worded with a statement "What are the weights needed to weigh any weight upto 100kgs such that minimum amount of iron is needed ; when you are given a pair of scales?" then the answer 1,3,6,10,30,60 would have been the best answer. The answer inspired by the triangle series.

This speaks the importance of the triangle series. And the rational growth expression in Result 1 matches with the triangle series 100 %.

Result 2

From (B) we have 1/2, 2/3, 3/4, 4/5, 5/6 . . . . and so on.

If the first rational number in (B) is (n-1)/n; then the second rational number is n/(n+1). Here (n-1) is taken odd always.

Now,

$$n/(n+1) - (n-1)/n = 1/(n+1)^2 + 1/(n+1)^3 + 1/(n+1)^4 + ... (I)$$

(I) is a geometric progression series till infinity. Here (n-1) is taken odd always.

Also it has been noted that

$$1/n = 1/(n+1) + 1/(n+1)^2 + 1/(n+1)^3 + 1/(n+1)^4 + ... (J)$$

(J) is a geometric progression series till infinity. Here n can be anything.

Result 2 – Background

When I was trying to draw a pentagon with the help of a compass only then at that point of time the result of 4/5 – 3/4 was crucial. Then I discovered that

$$4/5 - 3/4 = 1/5^2 + 1/5^3 + 1/5^4 + ...$$

I discovered that the formula is not just true of 3,4 and 5 but any three consecutive numbers placed in the above manner, that is the Result 2.

Finally I drew the pentagon using just a compass. The formula was assuming the radius to be 1, $\sqrt{2} - (3/4)(\sqrt{3} - \sqrt{2})$ was the side of the pentagon.

So trying to draw a pentagon with just a compass led me to the discovery of another expression of the growth of the rational series.

Result 3

From (E) we have $1^2/2^2$, $3^2/4^2$, $5^2/6^2$, $7^2/8^2$ . . . . and so on.

If the first rational number in (E) is $n^2/(n+1)^2$, then the second rational number is $(n+2)^2/(n+3)^2$.

Then

$$(n+2)^2/(n+3)^2 - n^2/(n+1)^2 = \text{prime number} /(k \times 4)^2$$

Here n is taken odd always.

The number k is the number from the triangle series. The triangle series is (C) and that is 1, 3, 6, 10, 15, 21 . . . . and so on.

Here k is $(((n+3)/2)(((n+3)/2)-1)/2)$

The expression $(n+2)^2/(n+3)^2 - n^2/(n+1)^2$ can therefore be used as a prime number generator along with the triangular numbers. The number of prime number generated by the function used is about 30%. However, if we eliminate certain obvious non-primes by checking with certain prime numbers like 5, 11 . . . . and so on; then the results improve. And about 50-60% of the results are prime numbers.

Result 3 – Facts

Prime numbers are an important area of research in number theory. One needs a foundation from which one can make more theories on them. The rational series of this form may be considered as a springboard from where mathematicians can move ahead and form more difficult theories on prime numbers.

Result 4

From (F) we have $1^2/2^2$, $2^2/3^2$, $3^2/4^2$, $4^2/5^2$ . . . . and so on

If the first rational number in (F) is $(n-1)^2/(n)^2$, then the second rational number is

$(n)^2/(n+1)^2$.

Here n is taken even always.

Then

$(n)^2/(n+1)^2 - (n-1)^2/(n)^2 = 2/(n+1)^2 - 1/(n+1)^4 + 1/(2(n+1)^6) - 1/(4(n+1)^8) + ...$ to m terms. (K)

In (K) the RHS is a geometric progression series till m terms.

Result 5

From (E) we have $1^2/2^2$, $3^2/4^2$, $5^2/6^2$, $7^2/8^2$ . . . . and so on

If the first rational number in (E) is $(n-1)^2/(n)^2$, then the second rational number is

$(n+1)^2/(n+2)^2$.

Here n is taken even always.

Then

$(n+1)^2/(n+2)^2 - (n-1)^2/(n)^2 = 4/(n+1)^2 - 4/(n+1)^3 + 4/(n+1)^4 - 4/(n+1)^5$ . . to m terms.(L)

In (L) the RHS is a geometric progression series till m terms.

Result 6

From (G) we have $\sqrt{1}/\sqrt{2}$, $\sqrt{3}/\sqrt{4}$, $\sqrt{5}/\sqrt{6}$, $\sqrt{7}/\sqrt{8}$ . . . . . . and so on.

If the first number in (G) is $\sqrt{(n-2)}/\sqrt{(n-1)}$, then the second rational number is $\sqrt{(n)}/$

$\sqrt{(n+1)}$.

Here n is taken odd always.

Then

$\sqrt{(n)}/ \sqrt{(n+1)} - \sqrt{(n-2)}/ \sqrt{(n-1)} = 1/n^2 + 1/(2n^3)+1/(4n^4)+1/(8n^5) \ldots$ to m terms. (M)

In (M) the RHS is a geometric progression series till m terms.

Result 7

From (H) we have $\sqrt{1}/\sqrt{2}, \sqrt{2}/\sqrt{3}, \sqrt{3}/\sqrt{4}, \sqrt{4}/\sqrt{5}. \ldots$ and so on.

If the first number in (G) is $\sqrt{(n-1)}/ \sqrt{(n)}$, then the second rational number is $\sqrt{(n)}/ \sqrt{(n+1)}$.

Here n is taken even always.

Then

$\sqrt{(n)}/ \sqrt{(n+1)} - \sqrt{(n-1)}/ \sqrt{(n)} = 1/(2n^2) - 1/(4n^3) + 1/(8n^4)-1/(16n^5) \ldots$ to m terms.(N)

In (N) the RHS is a geometric progression series till m terms.

Result 4, 5, 6 and 7 – Facts

The R.H.S in the results 4,5,6 & 7 are each geometric series which are correct in usage only when used up to a suitable number of terms. Say m. When in the L.H.S the value of n increases then m too increases. Only digits upto 10 places of decimal have been tried in this whole paper. Digits more than 10 places of decimal have not been tried. This was because only the calculator app in an android app was used.

An example as a prime number generator

Putting the results Result 3, Result 4, Result 5, Result 6 and Result 7 an algorithm is made for generating prime number. The results are quite good in some cases while in other it is not. However by suffixing and prefixing suitably with odd numbers 1,3,7 and 9 for suffixes and 1,2,3,4,5,6,7,8 and 9 for prefixes more prime numbers can be generated in almost all cases. The algorithms are

I.   Prime number = $n^2(n^2/4 + n + 1)(4/n^2 - - 4/n^3 + 4/n^4 - 4/n^5 \ldots \ldots)$
II.   Prime number = $n^2(n^2/4 + n + 1)(1/n^2 + 1/(2n^3)+1/(4n^4)+1/(8n^5) \ldots)$
III.   Prime number = $n^2(n^2/4 + n + 1)(2/n^2 - 1/n^4 +1/2n^6 - 1/(4n^8 + \ldots)$
IV.   Prime number = $n^2(n^2/4 + n + 1)(1/(2n^2) - 1/(4n^3) + 1/(8n^4)-1/(16n^5) \ldots)$

n is even in all the above.

The above formulas are using all the results obtained in Result 3, Result 4, Result 5, Result 6 and Result 7. In this method when an even number is obtained then the nearest odd number is taken. Let us consider n = 600. Using 600 in the above i), ii), iii) and iv) the following results are obtained:-

I. 361799

II. 90677

III. 181201

IV. 45263

All the above are prime in the case of n = 600. However there are cases where such is not the case.

In the Result 1 – Background , an accepted solution to the puzzle was 1,3,6,10, 30 & 60. In this 0 was added to the triangle numbers 1,3 and 6.

Similarly by adding suitable odd numbers in suffix to 361799, 90677, 181201 and 45263 a lot of other prime numbers can be generated. E.g. 3617993, 906779, 452633 etc.

Also by adding suitable prefix a lot of other prime numbers can be generated. E.g. 6361799, 290677, 9181201,345263 etc.

Conclusion

The rational series of numbers are a series which can be expressed in terms of geometric progression till infinity (Result 2) or as a geometric progression till a finite number of terms (Result 4, Result5, Result 6 and Result 7).

If we use these series appropriately, then inferences on prime numbers can also be drawn. From the above example it is clear that prime numbers can be generated with the algorithms based on Rational Series. Though there is a necessity that results be tested for prime, nevertheless the fact cannot be denied that fewer steps are needed to test for prime numbers if we use this method.

References

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