International Research Journal of Mathematics, Engineering and IT
ISSN: (2349-0322)
Association of Academic Researchers and Faculties (AARF)
Impact Factor- 8.132 Volume 13, Issue 04, April 2026
Website- www.aarf.asia, **Email** : editoraarf@gmail.com

Analytical Solutions and Qualitative Analysis of Ordinary Differential Equations

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DOI: aar.irjmeit.33989.22767

Abstract: This article provides a comprehensive overview of the methods and techniques for solving second-order ordinary differential equations when there are constant coefficients. The paper provides a detailed analysis of various methods for solving second-order ODEs (ordinary differential equations) with constant coefficients and groups solutions based on the roots of the characteristic equation. Repeating roots, complex conjugate roots, and various real roots are the three primary types of roots covered here. When the real roots are different, the solution is a linear combination of exponential functions. The goal of this study is to create a clear and straightforward roadmap that will assist researchers and students who are interested in understanding and solving 2nd order ODE amid invariable coefficients.In the case of repeated roots, the problem is solved by combining exponential functions with a linear term. The purpose of this essay is to provide a concise and understandable guidance for academics and students interested in this important topic.

Keywords: Second-order differential equations, ordinary differential equations, exponential functions, second- order homogeneous

1. Introduction

An equation that connects a function to one or additional of its derivatives is called a differential equation. The function is unknown. If function is univariate more specifically, if its province is a linked division of R, then the differential equation is considered ordinary. There are numerous applications for ordinary differential equations. These various contexts include population and growth models, as well as the basic laws of physics, mechanics, electricity, and thermodynamics. Ordinary differential equations (ODEs) occur in a variety of mathematical, social, and natural scientific contexts. For instance, the self-adjoint Legendre Differential in physics the solutions of the wave functions of hydrogen atoms and the angular momentum in single particle quantum mechanics give rise to ordinary differential equations.Their answers comprise the polar angle part of the spherical harmonics foundation of the multipole expansion used in electromagnetic and gravitational statics. For example, in engineering, several difficult problems in the domains of static and dynamic mechanics can be addressed by solving self-adjoint Bessel equations. Multiple differentials, derivatives, and functions are connected to form differential equations. It is common for quantities to appear in differential equations as gradients of other quantities or as the rate at which other quantities change.

Differential equations are, as is widely known, one of the most important mathematical tools for building models in many different domains, such as engineering, mathematics, physics, elasticity, dynamics, chemistry, and many more.A renewed interest in ODE theory has been spurred by the development of dynamical systems, nonlinear analysis, and its submission in knowledge and engineering over the past few decades. Differential equations and mathematical modeling can be used to study a wide range of social issues. Among the topics that clearly fit the math in an ODE course are aspects of population problems, such as population growth, overpopulation, carrying capacity of an ecology, the effect of yield, such as fish or hunt, on a populace and how overharvesting can lead to species extinction, and interactions between multiple species populations, such as predator-prey, supportive, and action-ready species.In many additional application disciplines, such as population dynamics, electronic circuits, molecular dynamics, chemical impact kinetics, and technicalities, ordinary differential equations are utilized extensively in science and engineering. Two of the most common specific areas that require differential equation modeling are geometry and analytical mechanics. Numerous scientific fields include physics and astronomy (celestial mechanics), chemistry (reaction rates), economics (stock trend, attention rates, and change in souk equilibrium prices), biology (genetic variation, infectious diseases), ecology and inhabitants modeling (population opposition), and meteorology (weather modeling). We can predict most natural phenomena with reasonable accuracy by solving differential equations.The question at hand is which strategy is the simplest and which method or methods are necessary to solve an ODE. What is the precision of the process? All of these factors need to be taken into account before solving an ODE, and this research will help by offering solutions.

The different accessible approaches are grouped based on different case usages and their relative effectiveness is compared to make it easy for a beginner mathematician or future scholars to choose a method to use when they encounter a differential equation. In this essay, we want to provide a comprehensive and detailed analysis of the methods used to solve this class of equations.The Standardized format of the homogeneous linear 2nd order ODE by constant coefficients utilized throughout the equation is what we begin by presenting. We explain what constant coefficients are and why they are important for solving these equations. Next, we offer the characteristic equation and its roots, which provide information on the nature of the solutions. We discuss the three situations: actual and independent roots, complex conjugate roots, and recurring roots. We develop the general answer for each circumstance and illustrate its application with examples. One of the good things about this article is that it covers both the mathematical and physical interpretations and applications of these equations.We discuss the relevance of the solutions to physical systems and their application to the analysis and prediction of their behavior. We provide examples from engineering and science to illustrate the applications of differential equations with constant coefficients throughout the equation. The methods and techniques covered in this article are essential to many academic fields, including engineering, physics, and mathematics. The article aims to provide a comprehensive guide for academics and students interested in this topic and could be used as a reference for addressing problems in connected fields. Solving these equations is therefore a vital task for numerous researchers and students across a range of academic fields.

2. Objectives of the study

- To determine how to solve ordinary differential equations analytically and collect qualitative information about the problem
- To determine a second-order ODE with constant coefficients.

3. Problem in hand

Many studies have been conducted on the solution of second-order differential equations. When the equation is linear, it is not a major problem because it can be solved analytically. Unfortunately, most interesting differential equations that emerge from modeling real-world problems are non-linear, and therefore very challenging to solve analytically. As a result, numerical techniques were developed and are wonderfully prepared to assist in solving those ODEs. Many computer programs have also been developed to help users solve these kinds of equations. The mathematical models of second-order differential equations are widely employed in many branches of science, engineering, and economics.Since these equations rarely have closed-form solutions, approximate solutions are usually found using numerical techniques. The many investigative and numerical techniques for solving second-order DE in low error limits, as well as computer-enhanced techniques for cracking evils that would otherwise be extremely challenging or time-consuming, will be examined and evaluated in this paper.

4. Literature of review

ODE, related integral equations (IE), and integro differential equations (IDE) have been used to model physical development since the establishment of separation and integration. Thanks to the development of modern computer capabilities, it is now possible to solve complex ODE, IE, and IDE models numerically with a high degree of accuracy. In 2025, Bouchenak and colleagues (2025) in this study, we discuss a novel modified conformable operator. Such an operator makes fractional calculus easy to learn because it yields precise results and satisfies the majority of the characteristics of the conventional derivative. There are two reasons why differential equations of this type are significant. To begin, they often show up in applications. Second, it is not too hard to find simple sets of solutions to these equations.We'll also look at the related fractional Cauchy-Euler type equation, which is used in engineering, physics, and other fields. Finally, we will illustrate the process with some numerical examples of the previously mentioned class of fractional differential equations.

In this work, Rivera-Rebolledo and Rivera-Figueroa (2015) present a straightforward method for resolving second-order linear differential equations with non-homogeneous constant coefficients. One advantage of this method is that it does not require the existence and uniqueness theorems to solve the starting values problem. This method also has the advantage of producing a single formula for the general solution, which implies that it represents the general solution independent of the properties of the roots of the characteristic problem.Stated differently, it is irrelevant whether the roots are equal or different real numbers or two conjugated complex numbers.

In 2021, Lozada and companions (2021) An summary of the present literature in the field, assistance in integrating existing information, and identification of potential roadblocks and points of view for further study on the topic are the three objectives of this paper's appraisal of the literature on ODE lessons and learning. We employed a methodology that blended a comprehensive literature review with bibliometric analysis.Highlighting the most recent research findings in this area, outlining the current research directions in the teaching and erudition of ODE, outlining the number of issues that will be covered in the coming years, and providing a starting point for scholars who want to follow this line of inquiry are the main contributions of the paper.

Hasan and Zhu (2009) This paper presents an efficient adaption of the Adomian decomposition approach to solve second-order ODE via stable coefficients. The proposed method can be applied to both linear and nonlinear situations. A few examples were given to show that the method may be applied to both linear and nonlinear ordinary differential equations.

Linot and colleagues (2023) when modeling spatiotemporal processes using data, it is often necessary to carefully consider the dynamics of the high wave numbers. This task gets extremely challenging when the system of interest exhibits chaotic dynamics or shocks. Furthermore, we find that stabilized neural ODE models are far more robust to noisy initial conditions than the traditional neural ODE approach. In this case, none of these results can be obtained using standard neural ODEs.

In Nair and associates (2025) The increase in published material over the past decade indicates the tremendous growth of the discipline of neural differential equations. This technology has several advantages, including the ability to mimic complex, nonlinear systems and the potential for extrapolation beyond the observable data.Neural network topologies utilized in NDEs, training strategies, and applications across multiple domains are covered in this study. By analyzing and categorizing earlier research, the current work offers guidance for mathematicians, computer scientists, and engineers. It also discusses the advantages of NDEs as well as some of the current problems and study areas in the field.

Fröhlich and Sorger (2022) Ordinary differential equation (ODE) models are widely used to analyze biochemical reactions in cellular networks because they accurately describe the temporal evolution of these networks using mass action kinetics. Since the parameters of these models are rarely known a priori, they must frequently be determined via calibration using experimental data. Optimization-based calibration of ODE models is often challenging, even for low-dimensional scenarios.To make it easier to evaluate different approaches to ODE mock-up calibration, a variety of Hessian approximation plans are included. We evaluated fides on a recently developed corpus of biologically plausible benchmark challenges for which experimental evidence is available. Surprisingly, we discovered that optimizer performance varied significantly amongst implementations of the same mathematical instructions (algorithms). An analysis of possible reasons for poor optimizer performance highlights the limitations of the widely used Gauss-Newton, BFGS, and SR1 Hessian approximation approaches.

Singh and Ujlayan (2021) In this section of Resonance, we invite readers to pose issues that might arise in a classroom. We may solicit feedback, provide answers, or do both. In addition to discussing more general subjects, the "classroom" provides a forum for individual experiences and viewpoints on matters related to scientific education. We will also solve some well-known and unusual ODEs.

In this work, Semnani and He (2024) use these recently constructed neural differential equations to continuously model time series, demonstrating the robustness of neural differential equations in modeling elastoplastic path-dependent material behavior. We develop a unique sequential model, INCDE, for general time-variant dynamical systems, including path-dependent constitutive models.The INCDE is analyzed in terms of stability and convergence. Using a finite element process with various monotonic loading processes, demonstrate the robustness, accuracy, and consistency of the proposed approach.

In Turab and companions (2024) this paper thoroughly investigates second-order Ordinary Differential Equations (ODEs), with an emphasis on analytical and computational concerns, in order to study animal avoidance behaviors. Using Picard-Lindelöf and fixed-point theorems, we show that only one of its kind solutions is continuous and evaluate their stability using the Ulam-Hyers principle. In this work, a mass-spring-damper organization is mathematically approximated using the Runge-Kutta fourth-order (RK4) and Euler methods.A detailed analysis of the arithmetic methodology, which includes a detailed evaluation of in collaboration complete and relative blunders, shows how effective these methods are in relation to the exact answers.

Turab and Sintunavarat (2021) The study of iterative DE is often associated with the many applications of calculus that form the foundation of the mathematical sciences. Using the Banach fixed point theorem, this work investigates a particular class of second-order iterative differential equations and establishes the exclusivity and continuity of the proposed DE solution. We investigate the Hyers-Ulam-Rassias sort solidity of an explanation to the future iterative boundary-value problem, and provide three illustrated examples to support our main findings.

5. Results and Data Analysis

Analytical methods, as their name suggests, involve "analyzing" the differential equation to make it easily recognizable or to create a precise differential on a single side. Analysis enables us to reduce the arrangement to a single, more easily cracked equation for higher order problems. Even though analytical processes are highly capable of providing reliable results, they have several limitations, including the complexity of psychiatry and the fact that not all DEs can be solved using them. The type and structure of differential equations determine the distinct analytical systems. I describe here the different analytical methods and the kinds of ODEs they can solve.This section looks with the particular case of 2nd order homogeneous linear DE, in which every coefficient is a real constant. Stated otherwise, we shall concentrate on the equation. This section focuses on the 2nd order homogeneous LDE, where each coefficient has a real value. In other words, we shall only examine equations that resemble this:

$$b_0(y)\frac{d^2x}{dy^2}+b_1(y)\frac{dx}{dy}+b_2(y)x=0 \dots\dots (1)$$

In order to find solutions to equation (1) in the form of $x=e^{ny}$, we shall select the constant m so that e^{ny}, satisfies the equation. We can write: $x=e^{ny}$, assuming that it is a solution for a given value of n.

$$\frac{dx}{dy}=ne^{ny},\frac{d^2x}{dy^2}=n^2e^{ny}\dots\dots(2)$$

When we substitute in (1), we get

$$b_0(y)n^2e^{ny}+b_1(y)ne^{ny}+b_2(y)e^{ny}=0\dots\dots(3)$$

We can obtain a polynomial equation in the variable m since $e^{ny}\neq 0$.

$$b_0(y)n^2+b_1(y)n+b_2(y)=0 \dots\dots (4)$$

The aforementioned equation is known as the differential equation (1) characteristic equation or auxiliary equation. The following three situations could occur when resolving the auxiliary equation:

Diverse Actual Origins

If the roots of (4) are different for n_1 and n_2, then

$$x=e^{n_1y},x=e^{n_2y}\dots\dots.(5)$$

are separate answers to (1). Consequently, this is the generic answer to (1).

$$x=d_1e^{n_1y}+d_2e^{n_2y}\dots\dots.(6)$$

Everywhere$\boldsymbol{d}_1,\boldsymbol{d}_2$ are random constants.

Example 1: If

$$2\frac{d^2x}{dy^2}-12\frac{dx}{dy}+16x=0\dots\dots(7)$$

The auxiliary formula is

$$2n^2-12n+16=0\dots..(8)$$

and so,

$$(n-4)(2n-4)=0$$

$$n = 4, 2$$

The roots are real and distinct. The equation's solutions are thus e^{4y} and e^{2y}, and the general solution can be written as follows:

$$x = d_1e^{4y} + d_2e^{2y} \text{.....(9)}$$

Where the constants d_1 and d_2 are arbitrary

Real Roots Repeated

The general solution of (1) can be written as follows if the auxiliary equation (4) contains repeated real roots that are distinct:

$$x = d_1e^{ny} + d_2ye^{ny} \text{......(10)}$$

Example 2: If

$$\frac{d^2x}{dy^2} - 4\frac{dx}{dy} + 4x = 0 \text{......(11)}$$

The auxiliary formula is

$$n^2 - 4n + 4 = 0 \text{...(12)}$$

or else,

$$f(b) = 0 \text{.....(13)}$$

This equation's roots are

$$n = 2, 2$$

The roots are real and distinct. The equation's solutions are thus e^{2y} and e^{2y}, and the general solution can be written as follows:

$$x = d_1e^{2y} + d_2ye^{2y} \text{.....(14)}$$

Where the constants d_1 and d_2 are arbitrary

Combine Complicated Roots

Since the coefficients in the auxiliary equation are real, we may conclude that the conjugate complex number, $b - ia$, is likewise a non-repeated root if the auxiliary equation has a non-repeated complex number root of the form $b + ia$. As a result, the general solution's matching component is:

$$x = d_1e^{(b+ia)y} + d_2\mathrm{e}^{(b-\mathrm{ia})\mathrm{y}}$$

$$x = e^{by}\left[d_1e^{iay} + d_2\mathrm{e}^{-\mathrm{iay}}\right]$$

$$= e^{by}[d_1(\cos ay + \mathrm{i}\sin ay) + d_2(\cos ay - i\mathrm{sinay})]$$

$$= e^{by}[(d_1 + d_2)\mathrm{cosay} + (d_1 - d_2)\mathrm{isinay})]$$

$$= e^{by}[B\mathrm{cosay} + A\mathrm{sinay})] \text{.......(15)}$$

Example 3: If

$$\frac{d^2x}{dy^2} - 6\frac{dx}{dy} + 25x = 0\text{….(16)}$$

The auxiliary formula is

$$n^2 - 6n + 25 = 0\text{….(17)}$$

When we solve it, we discover

$$n = \frac{-6 \pm \sqrt{36 - 100}}{2} = \frac{6 \pm 8i}{2} = 3 \pm 4i\text{….(18)}$$

The conjugate complex numbers $b \pm ai$, where b = 3,a = 4, are the roots in this case. The solution can be expressed generally as

$$x = e^{3y}(d_1 \sin 4y + d_2 \cos 4y)\text{…..(19)}$$

Let us examine the non-uniform DE.

$$b_0(y)\frac{d^2x}{dy^2} + a_1(y)\frac{dx}{dy} + a_2(y)x = F(y) \dots \dots (20)$$

where the no homogeneous factor F is often a non-constant function of y, while the coefficients a_0, a_1 and a_2 are constants. The general solution of the aforementioned equation can be expressed as follows: $x = x_c + x_p$, where x_p is the complementary function and $x_c$is the general solution of the related homogeneous equation (1) with F substituted for zero. This solution is devoid of any arbitrary constants. Conversely, a specific integral is any solution to equation (1) that is devoid of arbitrary constants.

If the polynomial in yis

F(y)= y,

then

$$x_p = \frac{1}{f(C)}Y = [f(C)]^{-1}Y\text{……(21)}$$

This can be done by multiplying term by term and using the binomial expansion $[f(C)]^{-1}$. Partial fractions are occasionally used to make the expansions.

Example 4: If

$$\frac{d^2x}{dy^2} + 3\frac{dx}{dy} + 2x = 4y + 5\text{….(22)}$$

The auxiliary formula is

$$n^2 + 3n + 2 = 0$$

$$n = -2, -1$$

The roots are real and distinct. Therefore, the solutions that fulfill the equation are e^{-y} and e^{-2y}, and the complementary solution is the one that is not related to any specific initial condition.

$$x_d = d_1e^{-y} + d_2e^{-2y}\ldots.(23)$$

The specific remedy is,

$$x_p = By + A\ldots.(24)$$

where B and A are constant, unknown coefficients that need to be found. Calculating the equation's derivative results in:

$$x_p' = B; x_p'' = 0\ldots.(25)$$

When we replace these in the equation, we get

$$0 + 3(B) + 2(By + A) = 4y + 5\ldots.(26)$$

$$3B + 2A = 5 \text{or} 2B = 4\ldots.(27)$$

When we solve this, we obtain

$$B = 2 \text{ and } A = \frac{-1}{2}\ldots..(28)$$

By replacing these, we get

$$x_p = 2y - \frac{1}{2}\ldots..(29)$$

The generic solution can be expressed as follows:

$$x = x_d + x_p$$

$$x = d_1e^{-y} + \mathrm{d}e^{-2y} + 2y - \frac{1}{2}\ldots..(30)$$

Condition If

$F(y) = e^{by}$ is a stable, then $x_p = \frac{e^{by}}{f(b)}$, make available$f(b) \neq 0$,

Example 5: If

$$\frac{d^2x}{dy^2} + 6\frac{dx}{dy} + 8x = e^{4y}\ldots\ldots(31)$$

The auxiliary formula is

$$n^2 + 6n + 8 = 0$$

$$n_1 = -4, n_2 = -2\ldots.(32)$$

The roots are real and distinct. Therefore, the solutions that fulfill the equation are e^{-y} and e^{-2y}, and the complementary solution is the one that is not related to any specific initial condition.

$$x_d = d_1e^{-4y} + d_2e^{-2y}\ldots(33)$$

The specific remedy is,

$$x_p = Be^{4y}\ldots(34)$$

Calculating the equation's derivative results in:

$$x_p^{'} = 4Be^{4y}$$

$$x_p^{''} = 16Be^{4y} \ldots\text{.(35)}$$

By replacing these, we get

$$16Be^{4y} + 6(4Be^{4y}) + 8Be^{4y} = e^{4y}$$

$$48Be^{y} = e^{4y}$$

$$48B = 1$$

$$B = \frac{1}{48} \ldots\text{.(36)}$$

By replacing these, we get

$$x_p = \frac{1}{48} e^{4y} \ldots\text{.(37)}$$

The generic solution can be expressed as follows:

$$x = x_d + x_p \ldots\text{..(38)}$$

after that

$$x = d_1 e^{-4y} + d_2 e^{-2y} + \frac{1}{48} e^{4y} \ldots\ldots\text{.(39)}$$

Proviso

$$F(y) = \sin y \text{ or } \cos y \ldots\ldots\text{(40)}$$

subsequently

$$\frac{1}{f(C)^2} \sin by = \frac{1}{f(-b^2)} \sin by \ldots\text{.(41)}$$

in addition to

$$\frac{1}{f(b^2)} \cos by = \frac{1}{f(-b^2)} \cos by \ldots\text{.(42)}$$

with the exception of when

$$f(-b^2) = 0 \ldots\text{.(43)}$$

We distinguish,

$$\sin by = \sin by$$

$$C(\sin by) = b\cos by$$

$$C^{\,2} \, (\,\sin by) = -b^{\,2} \sin by$$

$$C^{\,3} \, (\sin by) = -b^{\,3} \cos by \ldots\text{.(44)}$$

in the same way

$$(C^{\,2})^m \sin by = (-b^{\,2})^{\,m} \sin by \ldots\text{.(45)}$$

in consequence

$$f(C^{\,2}\,) \sin by = f(-b^2) \sin by \ldots\text{.(46)}$$

in use by $\frac{1}{f(C)^2}$ on in cooperation sides, we acquire

$$\frac{1}{f(C^2)} f(C^2) \sin by = \frac{1}{f(C^2)} f(-b^2) \sin by \ldots\text{.(47)}$$

otherwise

$$f(-b^2) \neq 0....(48)$$

isolating by

$$f(-b^2),......(49)$$

we acquire

$$\frac{1}{f(C^2)}\sin by = \frac{1}{f(-b^2)}\sin by,.....(50)$$

make available

$$f(-b^2) \neq 0,.....(51)$$

in the same way

$$\frac{1}{f(C^2)}\cos by = \frac{1}{f(-b^2)}\cos by....(52)$$

Example 6: If

$$\frac{d^2x}{dy^2} + 3\frac{dx}{dy} + 2x = \cos 2y.....(53)$$

The auxiliary formula is

$$n^2 + 3n + 2 = 0$$

$$n_1 = -1, n_2 = -2....(54)$$

The roots are real and distinct. Therefore, the solutions that fulfill the equation are e^{-y} and e^{-2y}, and the complementary solution is the one that is not related to any specific initial condition.

$$x_d = d_1e^{-y} + d_2e^{-2y}.....(55)$$

The specific remedy is,

$$x_p = B\cos 2y + A\sin 2y.....(56)$$

Calculating the equation's derivative results in:

$$x_p^{'} = -2B\sin 2y + 2A\cos 2y$$

$$x_p^{''} = -4B\cos 2y - 4A\sin 2y......(57)$$

By replacing these, we get

$$-4B\cos 2y - 4A\sin 2y + 3(-2B\sin 2y + 2A\cos 2y) +$$

$$2(B\cos 2y + A\sin 2y)$$

$$= \cos 2y....(58)$$

By replacing these, we get

$$-2B + 6A = 1 - 6B - 2A = 0.....(59)$$

The generic solution can be expressed as follows:

$$B = \frac{-1}{20}, A = \frac{3}{20}$$

$$x_p = \frac{-1}{20}\cos 2y + \frac{3}{20}\sin 2y \ldots\ldots(60)$$

The generic solution can be expressed as follows:

$$x = x_d + x_p \ldots.(61)$$

$$x = d_1 e^{-y} + d_2 e^{-2y} + \frac{3}{20}\sin 2y - \frac{1}{20}\cos 2y \ldots.(62)$$

6. Conclusion

In conclusion, the solution of second order ODEs with constant coefficients is an important topic in mathematics, physics, and engineering. This family of equations is used in many different fields of study and has several applications in physical systems. The characteristic equation and its roots, as well as the three possible situations of distinct and real roots, complicated conjugate roots, and repeated roots, have all been covered in this article's comprehensive and in-depth study of the methods for solving these equations.We have also focused on the physical interpretations and applications of the answers to the analysis and prediction of the behavior of physical systems. This page is a helpful tool for academics and students who are interested in the topic; it can be used as a guide to solve problems in related fields.

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