



COLORING OF INTUITIONISTIC FUZZY GRAPH USING (α, β) -CUTS

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ABSTRACT

In this paper, the concept of Intuitionistic Fuzzy Graph Coloring using (α, β) -cuts are introduced with illustrative examples. Chromatic number, chromatic index and total chromatic number of a intuitionistic fuzzy graph using (α, β) -cuts are defined.

Key words: Chromatic number, Chromatic index, Total chromatic number, Intuitionistic fuzzy graph, (α, β) -cut.

1. Introduction:

Graph coloring has become a subject of great interest, largely because of its diverse theoretical results, unsolved problems, and numerous applications. Graph coloring dates back to 1852, when Francis Guthrie come up with the four color conjecture. A graph coloring is the assignment of a color to each of the vertices or edges or both in such a way that no two adjacent vertices and incident edges share the same color. Graph coloring has applications to many real world problems like scheduling, telecommunications and bioinformatics, etc.

The concept of fuzzy graph was introduced by Rosenfeld in 1975 and intuitionistic fuzzy sets [3] and intuitionistic fuzzy graph [2] were introduced by Krassimir T. Atanassov in 1986 and 1999 respectively. R.Parvathi et.al. discussed the intuitionistic fuzzy graph and its properties [6],[7]. The concept of chromatic number of fuzzy graph was introduced by Munoz et.al. [8]. The concept of fuzzy total coloring and fuzzy total chromatic number of a fuzzy graph were introduced by S.Lavanya et.al. [4]. V.Nivethana and A.Parvathi introduced fuzzy total coloring and chromatic number of a complete fuzzy graph [5]. Research in Intuitionistic Fuzzy graph theory and its applications have been increased considerably in recent years.

Intuitionistic Fuzzy graph model can represent a complex, imprecise and uncertain problem, where classical graph model may fail. In many cases, some aspects of a graph theoretic problem may be uncertain, for example, the vehicle travel time, or vehicle capacity on a road network may not be known exactly.

In this paper, we consider only the intuitionistic fuzzy graphs with crisp vertex set and intuitionistic fuzzy edge set. Also the concept of intuitionistic fuzzy vertex coloring, intuitionistic fuzzy edge coloring, intuitionistic fuzzy total coloring using (α, β) -cuts are

introduced with illustrative examples. Chromatic number, chromatic index, and total chromatic number of an intuitionistic fuzzy graph using (α, β) -cuts are defined.

2. Preliminaries

2.1. Definition

Let X be a non-empty set. Then a fuzzy set A in X (i.e., a fuzzy subset A of X) is characterized by a function of the form $\mu_A: X \rightarrow [0,1]$, such a function μ_A is called the membership function and for each $x \in X$, $\mu_A(x)$ is the degree of membership of x (membership grade of x) in the fuzzy set A .

In other words,

$$A = \{(x, \mu_A(x)) / x \in X\} \text{ where } \mu_A: X \rightarrow [0,1].$$

2.2. Definition

A fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma: V \rightarrow [0,1]$ and $\mu: V \times V \rightarrow [0,1]$, where for all $u, v \in V$, we have $\mu(u, v) \leq \sigma(u) \wedge \sigma(v)$.

2.3. Definition

An Intuitionistic Fuzzy set A in a set X is defined as an object of the form

$A = \{(x, \mu_A(x), \nu_A(x)) / x \in X\}$ where $\mu_A: X \rightarrow [0,1]$ and $\nu_A: X \rightarrow [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ respectively and for every $x \in X$; $0 \leq \mu_A(x) + \nu_A(x) \leq 1$.

2.4. Definition

Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$, where

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1: V \rightarrow [0,1]$ and $\nu_1: V \rightarrow [0,1]$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively and $0 \leq \mu_1(v_i) + \nu_1(v_i) \leq 1$, for every $v_i \in V$, ($i = 1, 2, \dots, n$).
- (ii) $E \subset V \times V$ where $\mu_2: V \times V \rightarrow [0,1]$ and $\nu_2: V \times V \rightarrow [0,1]$ are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$$

$$\nu_2(v_i, v_j) \geq \max[\nu_1(v_i), \nu_1(v_j)]$$

And $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Here the triplet $(v_i, \mu_{1i}, \nu_{1i})$ denote the vertex, the degree of membership and degree of non-membership of the vertex v_i . The triplet $(e_{ij}, \mu_{2ij}, \nu_{2ij})$ denote the edge, the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

2.5. Definition

A set of (α, β) -cut, generated by an Intuitionistic Fuzzy Set A , where $\alpha, \beta \in [0,1]$ are fixed numbers such that $\alpha + \beta \leq 1$, is defined as

$$A_{\alpha, \beta} = \{x \in X / \mu_A(x) \geq \alpha, \nu_A(x) \leq \beta, \quad \forall \alpha, \beta \in [0,1]\}$$

The family of (α, β) -cut sets $A_{\alpha, \beta}$ is monotone, that is for $\alpha, \beta, \gamma, \delta \in [0, 1]$ and $\alpha \leq \gamma, \beta \geq \delta$, We have $A_{\alpha, \beta} \supseteq A_{\gamma, \delta}$.

2.6. Definition [1]

The graph $G = (V, E)$ is a crisp graph, a coloring function is a mapping $C: V(G) \rightarrow N$ (where N is set of positive integers) such that $C(u) \neq C(v)$ if u and v are adjacent in G .

2.6. Definition [1]

The graph $G = (V, E)$ is a crisp graph, a k -coloring function is a mapping $C^k: V(G) \rightarrow \{1, 2, \dots, k\}$ such that $C^k(u) \neq C^k(v)$ if u and v are adjacent in G . A graph G is k -colorable if it admits k -coloring. The chromatic number $\chi(G)$, of a graph G is the minimum k for which G is k -colorable.

2.7. Definition (Munoz et.al.[8])

If $G = (V, \mu)$ is such a fuzzy graph where $V = \{1, 2, 3, \dots, n\}$ and μ is a fuzzy number on the set of all subsets of $V \times V$. Assume $I = A \cup \{0\}$ where $A = \{\alpha_1 < \alpha_2 < \dots < \alpha_k\}$ is the fundamental set (level set) of G . For each $\alpha \in I$, G_α denote the crisp graph $G_\alpha = (V, E_\alpha)$ where $E_\alpha = \{(i, j) / 1 \leq i < j \leq n, \mu(i, j) \geq \alpha\}$ and $\chi_\alpha = \chi(G_\alpha)$ denote the chromatic number of crisp graph G_α . By this definition the chromatic number of the fuzzy graph G is the fuzzy number $\chi(G) = \{(i, v(i)) / i \in X\}$ where $v(i) = \max\{\alpha \in I / i \in A_\alpha\}$ and $A_\alpha = \{1, \dots, \chi_\alpha\}$.

3. Intuitionistic fuzzy graphs with crisp vertices and intuitionistic fuzzy edges:

3.1. Definition

The graph $\hat{G} = (V, \hat{E})$ is a intuitionistic fuzzy graph with crisp vertices and intuitionistic fuzzy edges, where

- (i) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_1: V \rightarrow \{0, 1\}$ and $\nu_1: V \rightarrow \{0, 1\}$ denote the degrees of membership and non-membership of the element $v_i \in V$ respectively and $\mu_1(v_i) + \nu_1(v_i) = 1$, for every $v_i \in V, (i = 1, 2, \dots, n)$.
- (ii) $E \subset V \times V$ where $\mu_2: V \times V \rightarrow [0, 1]$ and $\nu_2: V \times V \rightarrow [0, 1]$ are such that

$$\mu_2(v_i, v_j) \leq \min[\mu_1(v_i), \mu_1(v_j)]$$

$$\nu_2(v_i, v_j) \geq \max[\nu_1(v_i), \nu_1(v_j)]$$

And $0 \leq \mu_2(v_i, v_j) + \nu_2(v_i, v_j) \leq 1$ for every $(v_i, v_j) \in E$.

Here the triplet $(e_{ij}, \mu_{2ij}, \nu_{2ij})$ denote the edge, the degree of membership and degree of non-membership of the edge relation $e_{ij} = (v_i, v_j)$ on $V \times V$.

4. Intuitionistic Fuzzy Vertex Coloring & Chromatic number

4.1. Definition

Let $\{G_{\alpha, \beta} = (V, E_{\alpha, \beta}) / \alpha, \beta \in [0, 1]\}$ be the family of (α, β) -cuts of \hat{G} , where the (α, β) -cut of a intuitionistic fuzzy graph is the crisp graph $G_{\alpha, \beta} = (V, E_{\alpha, \beta})$ with

$$E_{\alpha, \beta} = \{(i, j) / i, j \in V, \mu_{2ij} \geq \alpha, \nu_{2ij} \leq \beta\}.$$

Hence, to determine chromatic number of IFG \hat{G} , first step is to find the corresponding family of (α, β) -cuts of \hat{G} then determine the chromatic number $\chi_{\alpha, \beta}$ of each $G_{\alpha, \beta}$ by using crisp k- vertex coloring $C_{\alpha, \beta}^k$. The Chromatic number of \hat{G} is defined through a monotone family of sets.

4.2. Definition

For a intuitionistic fuzzy graph $\hat{G} = (V, \hat{E})$, its chromatic number is the intuitionistic fuzzy number $\chi(\hat{G}) = \{(x, m(x), n(x))/x \in X\}$, where $X = \{1, \dots, |V|\}$, $m(x) = \sup\{\alpha \in [0,1]/x \in A_{\alpha, \beta}\}$, $n(x) = \inf\{\beta \in [0,1]/x \in A_{\alpha, \beta}\}$, $x \in X$ and $A_{\alpha, \beta} = \{\chi_{1,0}, \dots, \chi_{\alpha, \beta}\}$, $\alpha, \beta \in [0,1]$.

The chromatic number of a intuitionistic fuzzy graph is a normalized intuitionistic fuzzy number whose modal value is associated with the empty edge set graph. Its meaning depends on the sense of index (α, β) . It can be interpreted that for lower values of α and higher values of β , there are many adjacent edges between the vertices so that more colors are needed in order to consider the incompatibilities.

At the same time, for higher values of α and lower values of β , there are fewer adjacent edges and less colors are needed. The intuitionistic fuzzy coloring problem consists of determining the chromatic number of a intuitionistic fuzzy graph and an associated coloring function. For any level (α, β) , the minimum number of colors needed to color the crisp graph $G_{\alpha, \beta}$ can be computed and hence chromatic number of an IFG is determined using its (α, β) -cuts.

4.3. Example

Consider the intuitionistic fuzzy graph $\hat{G} = (V, \hat{E})$ in figure 4.1, with six crisp vertices and eight intuitionistic fuzzy edges.

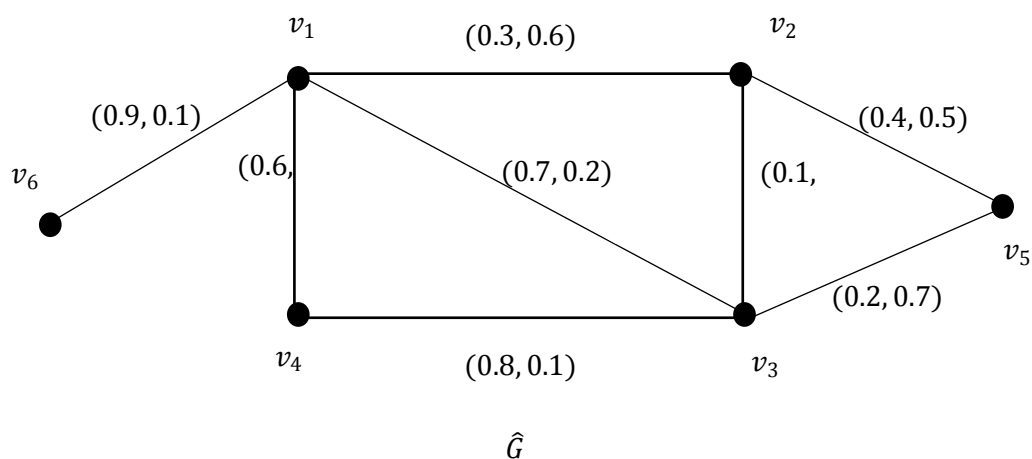
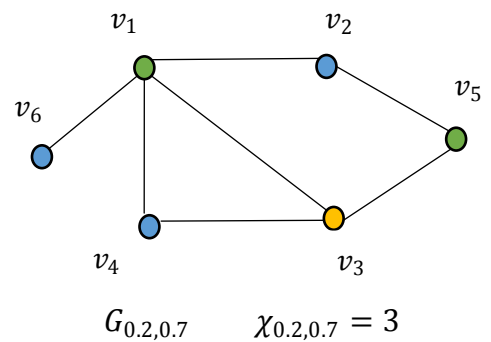
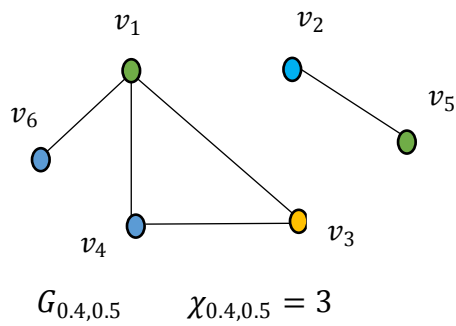
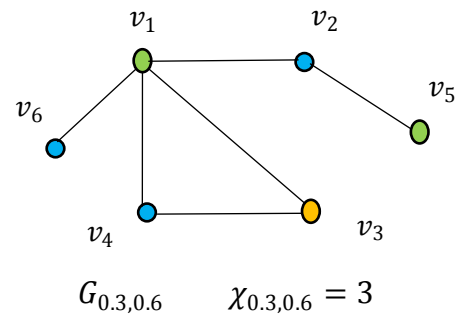
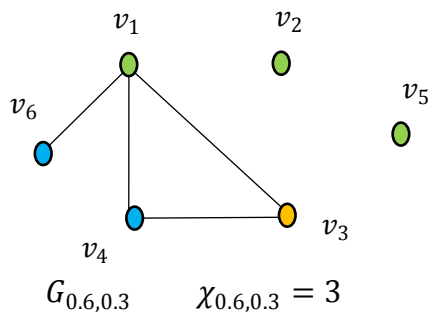
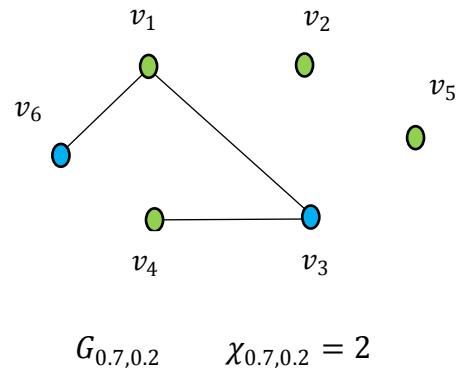
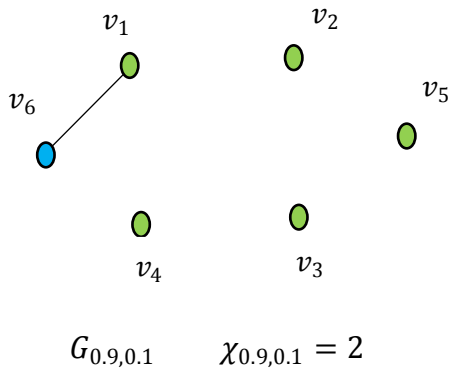
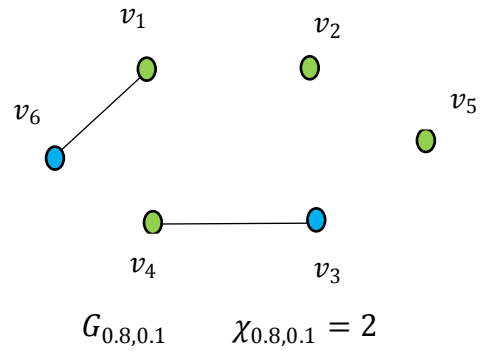
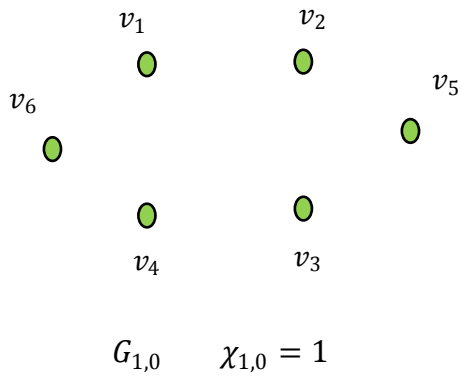


Figure 4.1

For the IFG in figure 4.1, there are ten crisp graphs $G_{\alpha, \beta} = (V, E_{\alpha, \beta})$ as shown in figure 4.2. For each $\alpha, \beta \in [0,1]$, the Table 4.1 contains the edge set $E_{\alpha, \beta}$, the chromatic number $\chi_{\alpha, \beta}$ and vertex coloring $C_{\alpha, \beta}^k$.



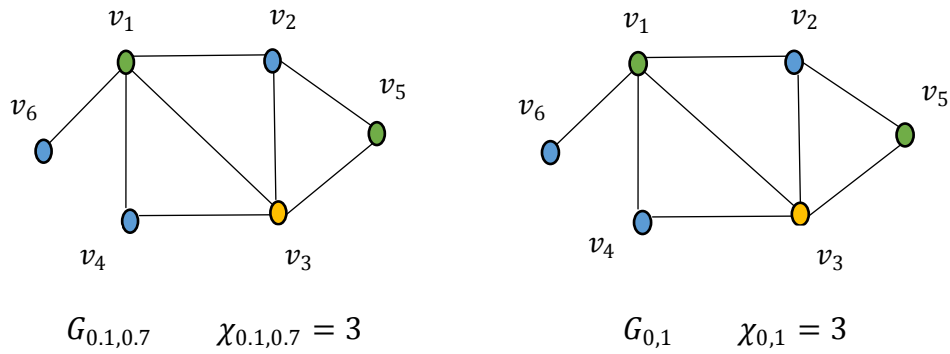


Figure 4.2

Table 4.1

(α, β)	$E_{\alpha, \beta}$	$\chi_{\alpha, \beta}$	$C_{\alpha, \beta}(v_1)$	$C_{\alpha, \beta}(v_2)$	$C_{\alpha, \beta}(v_3)$	$C_{\alpha, \beta}(v_4)$	$C_{\alpha, \beta}(v_5)$	$C_{\alpha, \beta}(v_6)$
(1,0)	\emptyset	1	1	1	1	1	1	1
(0.9,0.1)	$v_1 v_6$	2	1	1	1	1	1	2
(0.8,0.1)	$v_1 v_6, v_4 v_3$	2	1	1	2	1	1	2
(0.7,0.2)	$v_1 v_6, v_4 v_3, v_1 v_3$	2	1	1	2	1	1	2
(0.6,0.3)	$v_1 v_6, v_4 v_3, v_1 v_3$ $v_1 v_4$	3	1	1	3	2	1	2
(0.4,0.5)	$v_1 v_6, v_4 v_3, v_1 v_3$ $v_1 v_4, v_2 v_5$	3	1	2	3	2	1	2
(0.3,0.6)	$v_1 v_6, v_4 v_3, v_1 v_3$ $v_1 v_4, v_2 v_5, v_1 v_2$	3	1	2	3	2	1	2
(0.2,0.7)	$v_1 v_6, v_4 v_3, v_1 v_3$ $v_1 v_4, v_2 v_5, v_1 v_2$ $v_5 v_3$	3	1	2	3	2	1	2
(0.1,0.7) & (0,1)	$v_1 v_6, v_4 v_3, v_1 v_3$ $v_1 v_4, v_2 v_5, v_1 v_2$ $v_5 v_3, v_2 v_3$	3	1	2	3	2	1	2

The chromatic number of IFG \hat{G} is $\chi_{\alpha, \beta} = \{(1,1,0), (2,0.9,0.1), (3,0.6,0.3)\}$.

5. Intuitionistic Fuzzy Edge Coloring & Chromatic Index

5.1. Definition

In crisp case the edge chromatic number of a graph is either Δ or $\Delta + 1$, where Δ is the maximum vertex degree.

Hence, to determine chromatic index of IFG, \hat{G} , first step is to find the corresponding family of (α, β) -cuts of \hat{G} (ie., $G_{\alpha, \beta}$), then determine the chromatic index (edge chromatic number) $\chi'_{\alpha, \beta}$ of each $G_{\alpha, \beta}$ by using crisp k-edge coloring $C_{\alpha, \beta}^k$.

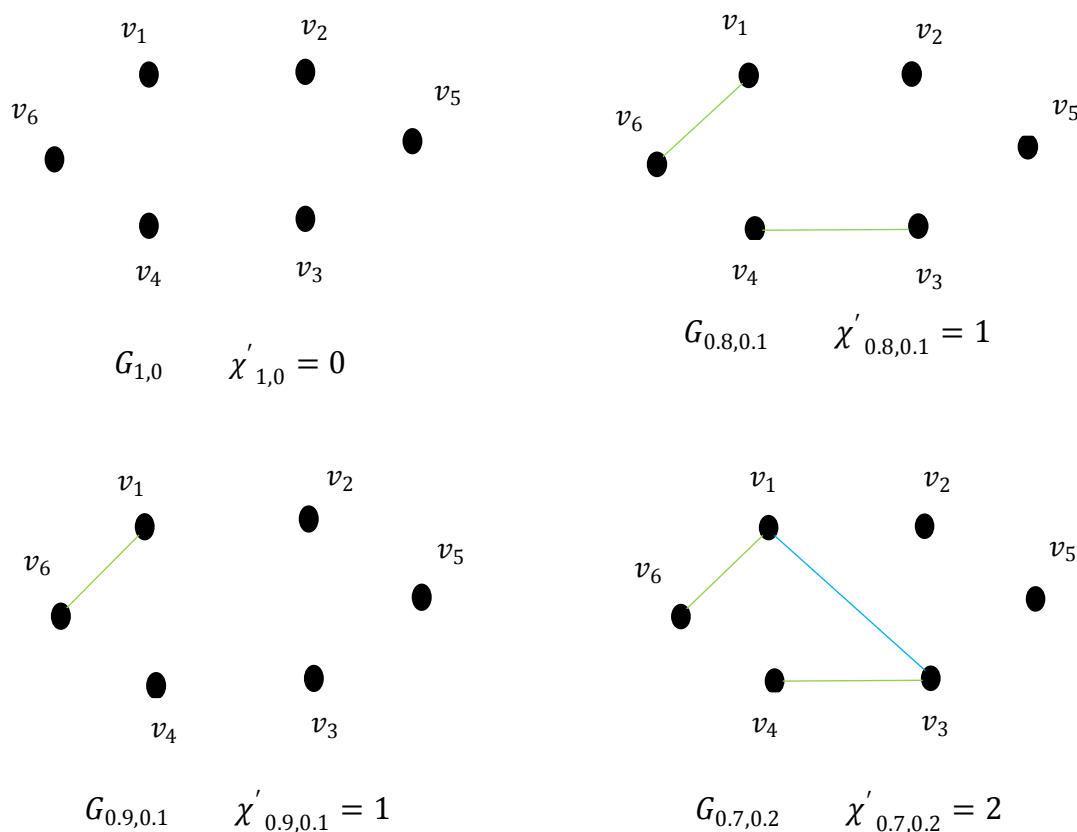
5.2. Definition

For a intuitionistic fuzzy graph $\hat{G} = (V, \hat{E})$, its edge chromatic number (chromatic index) is the intuitionistic fuzzy number $\chi'(\hat{G}) = \{(x, m(x), n(x))/x \in X\}$, where $X = \{1, \dots, \Delta + 1\}$, $m(x) = \sup\{\alpha \in [0, 1]/x \in A_{\alpha, \beta}\}$, $n(x) = \inf\{\beta \in [0, 1]/x \in A_{\alpha, \beta}\}$, $x \in X$ and $A_{\alpha, \beta} = \{\chi_{1,0}, \dots, \chi_{\alpha, \beta}\}$, $\alpha, \beta \in [0, 1]$.

5.3. Example

Consider the IFG $\hat{G} = (V, \hat{E})$, in Example 4.3.

There are ten crisp graphs $G_{\alpha, \beta} = (V, E_{\alpha, \beta})$ as shown in figure 5.1 for different values $\alpha, \beta \in [0, 1]$. For each $\alpha, \beta \in [0, 1]$, the Table 5.1 contains the edge set $E_{\alpha, \beta}$, the Edge chromatic number $\chi'_{\alpha, \beta}$ and edge coloring $C_{\alpha, \beta}^k$.



(α, β)	$E_{\alpha, \beta}$	$\chi'_{\alpha, \beta}$	$C_{\alpha, \beta}$	$C_{\alpha, \beta}$	$C_{\alpha, \beta}$	$C_{\alpha, \beta}$	$C_{\alpha, \beta}$	$C_{\alpha, \beta}$	$C_{\alpha, \beta}$	$C_{\alpha, \beta}$
			$(v_1 v_6)$	$(v_4 v_3)$	$(v_1 v_3)$	$(v_1 v_4)$	$(v_2 v_5)$	$(v_1 v_2)$	$(v_5 v_3)$	$(v_2 v_3)$

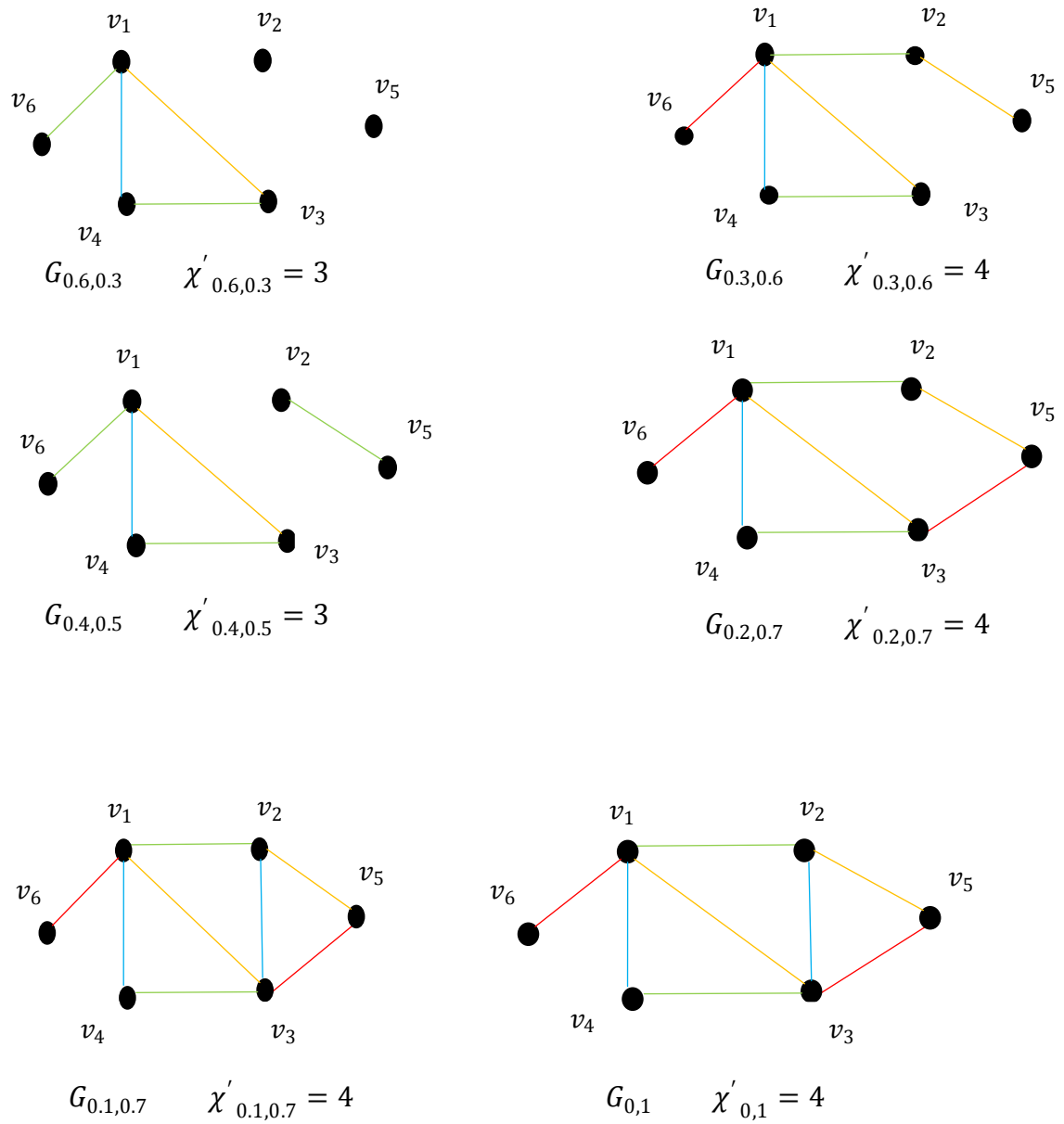


Figure 5.1

(1,0)	\emptyset	0	0	0	0	0	0	0	0	0
(0.9,0.1)	v_1v_6	1	1	0	0	0	0	0	0	0
(0.8,0.1)	v_1v_6, v_4v_3	1	1	1	0	0	0	0	0	0
(0.7,0.2)	v_1v_6, v_4v_3, v_1v_3	2	1	1	2	0	0	0	0	0
(0.6,0.3)	v_1v_6, v_4v_3, v_1v_3 v_1v_4	3	1	1	3	2	0	0	0	0
(0.4,0.5)	v_1v_6, v_4v_3, v_1v_3 v_1v_4, v_2v_5	3	1	1	3	2	1	0	0	0
(0.3,0.6)	v_1v_6, v_4v_3, v_1v_3 v_1v_4, v_2v_5, v_1v_2	4	4	1	3	2	3	1	0	0
(0.2,0.7)	v_1v_6, v_4v_3, v_1v_3 v_1v_4, v_2v_5, v_1v_2 v_5v_3	4	4	1	3	2	3	1	4	0
(0.1,0.7)	v_1v_6, v_4v_3, v_1v_3 v_1v_4, v_2v_5, v_1v_2 v_5v_3, v_2v_3	4	4	1	3	2	3	1	4	2
(0,1)	v_1v_6, v_4v_3, v_1v_3 v_1v_4, v_2v_5, v_1v_2 v_5v_3, v_2v_3	4	4	1	3	2	3	1	4	2

The edge chromatic number (chromatic index) of IFG \hat{G} is

$$\chi'_{\alpha,\beta} = \{(0,1,0), (1,0.9,0.1), (2,0.7,0.2), (3,0.6,0.3), (4,0.3,0.6)\}.$$

Table 5.1

6. Intuitionistic Fuzzy Total Coloring & Total chromatic number

6.1. Definition

In crisp case the total chromatic number of a graph is atmost $\Delta + 2$, (By total coloring conjecture) where Δ is the maximum vertex degree.

Hence, to determine total chromatic number of IFG, \hat{G} , first step is to find the corresponding family of (α, β) -cuts of \hat{G} (ie., $G_{\alpha, \beta}$), then determine the total chromatic number $\chi^T_{\alpha, \beta}$ of each $G_{\alpha, \beta}$ by using crisp k- total coloring $C_{\alpha, \beta}^k$.

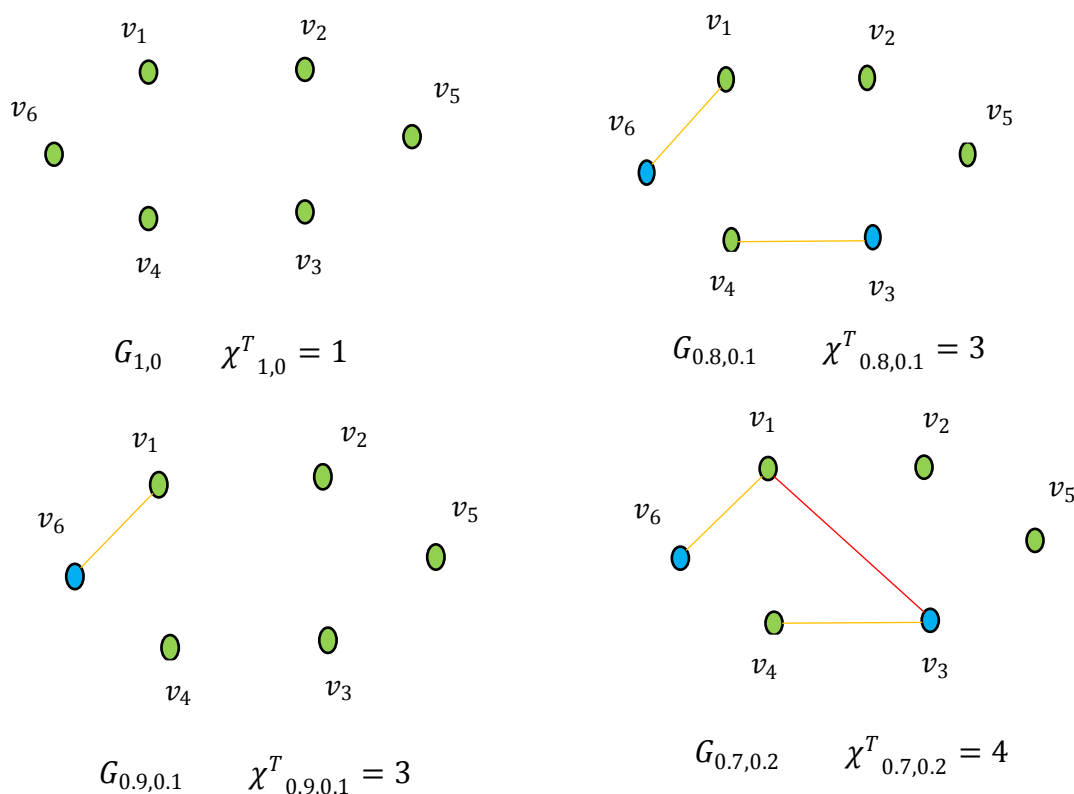
6.2. Definition

For a intuitionistic fuzzy graph $\hat{G} = (V, \hat{E})$, its total chromatic number is the intuitionistic fuzzy number $\chi^T(\hat{G}) = \{(x, m(x), n(x))/x \in X\}$, where $X = \{1, \dots, \Delta + 2\}$, $m(x) = \sup\{\alpha \in [0,1]/x \in A_{\alpha, \beta}\}$, $n(x) = \inf\{\beta \in [0,1]/x \in A_{\alpha, \beta}\}$, $x \in X$ and $A_{\alpha, \beta} = \{\chi_{1,0}, \dots, \chi_{\alpha, \beta}\}$, $\alpha, \beta \in [0,1]$.

6.3. Example

Consider the intuitionistic fuzzy graph $\hat{G} = (V, \hat{E})$, in Example 4.3.

There are ten crisp graphs $G_{\alpha, \beta} = (V, E_{\alpha, \beta})$ as shown in figure 6.1. For each $\alpha, \beta \in [0,1]$, the Table 6.1 contains the edge set $E_{\alpha, \beta}$, the total chromatic number $\chi^T_{\alpha, \beta}$ and total coloring $C_{\alpha, \beta}^k$.



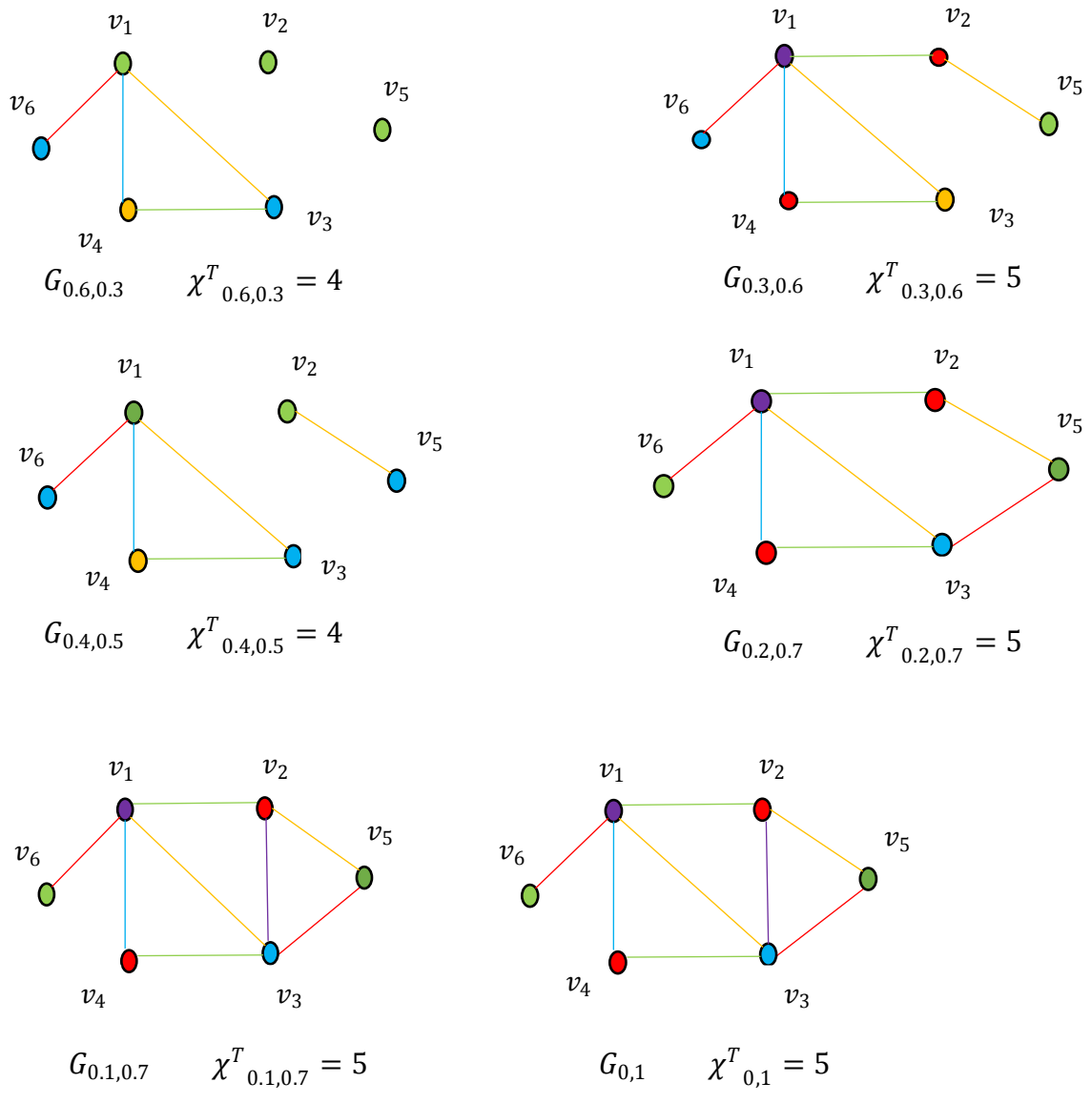


Figure 6.1

Table 6.1

(α, β)	$E_{\alpha, \beta}$	$\chi^T_{\alpha, \beta}$	$C_{\alpha, \beta}$ (v_1)	$C_{\alpha, \beta}$ (v_2)	$C_{\alpha, \beta}$ (v_3)	$C_{\alpha, \beta}$ (v_4)	$C_{\alpha, \beta}$ (v_5)	$C_{\alpha, \beta}$ (v_6)	$C_{\alpha, \beta}$ ($v_1 v_6$)	$C_{\alpha, \beta}$ ($v_4 v_3$)	$C_{\alpha, \beta}$ ($v_1 v_3$)	$C_{\alpha, \beta}$ ($v_1 v_4$)	$C_{\alpha, \beta}$ ($v_2 v_5$)	$C_{\alpha, \beta}$ ($v_1 v_2$)	C (v)
(1,0)	\emptyset	1	1	1	1	1	1	1	0	0	0	0	0	0	0
(0.9,0.1)	$v_1 v_6$	3	1	1	1	1	1	2	3	0	0	0	0	0	0
(0.8,0.1)	$v_1 v_6, v_4 v_3$	3	1	1	2	1	1	2	3	3	0	0	0	0	0
(0.7,0.2)	$v_1 v_6, v_4 v_3, v_1 v_3$	4	1	1	2	1	1	2	3	3	4	0	0	0	0
(0.6,0.3)	$v_1 v_6, v_4 v_3, v_1 v_3,$ $v_1 v_4$	4	1	1	2	3	1	2	4	1	3	2	0	0	0
(0.4,0.5)	$v_1 v_6, v_4 v_3, v_1 v_3,$ $v_1 v_4, v_2 v_5$	4	1	1	2	3	2	2	4	1	3	2	3	0	0
(0.3,0.6)	$v_1 v_6, v_4 v_3, v_1 v_3,$ $v_1 v_4, v_2 v_5, v_1 v_2$	5	5	4	2	4	1	1	4	1	3	2	3	1	0
(0.2,0.7)	$v_1 v_6, v_4 v_3, v_1 v_3,$ $v_1 v_4, v_2 v_5, v_1 v_2,$ $v_5 v_3$	5	5	4	2	4	1	1	4	1	3	2	3	1	4
(0.1,0.7)	$v_1 v_6, v_4 v_3, v_1 v_3,$ $v_1 v_4, v_2 v_5, v_1 v_2,$ $v_5 v_3, v_2 v_3$	5	5	4	2	4	1	1	4	1	3	2	3	1	4
(0,1)	$v_1 v_6, v_4 v_3, v_1 v_3,$ $v_1 v_4, v_2 v_5, v_1 v_2,$ $v_5 v_3, v_2 v_3$	5	5	4	2	4	1	1	4	1	3	2	3	1	4

The total chromatic number of IFG \hat{G} is

$$\chi_{\alpha,\beta}^T = \{(1,1,0), (3,0.9,0.1), (4,0.7,0.2), (5,0.3,0.6)\}.$$

7. Conclusion

In this paper, the concept of intuitionistic fuzzy vertex coloring, intuitionistic fuzzy edge coloring and intuitionistic fuzzy total coloring using (α, β) -cuts are introduced with illustrative examples. Chromatic number, chromatic index, and total chromatic number of a intuitionistic fuzzy graph are defined.

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